

# **Shadows**

**CS425: Computer Graphics I**

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<https://fmiranda.me>

# Overview

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- Shadow projection
- Shadow Mapping
- Shadow Volume
- Shadow Accumulation

# Shadows

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# Shadows



# Shadows

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# Shadows

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# Shadows

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# Shadows



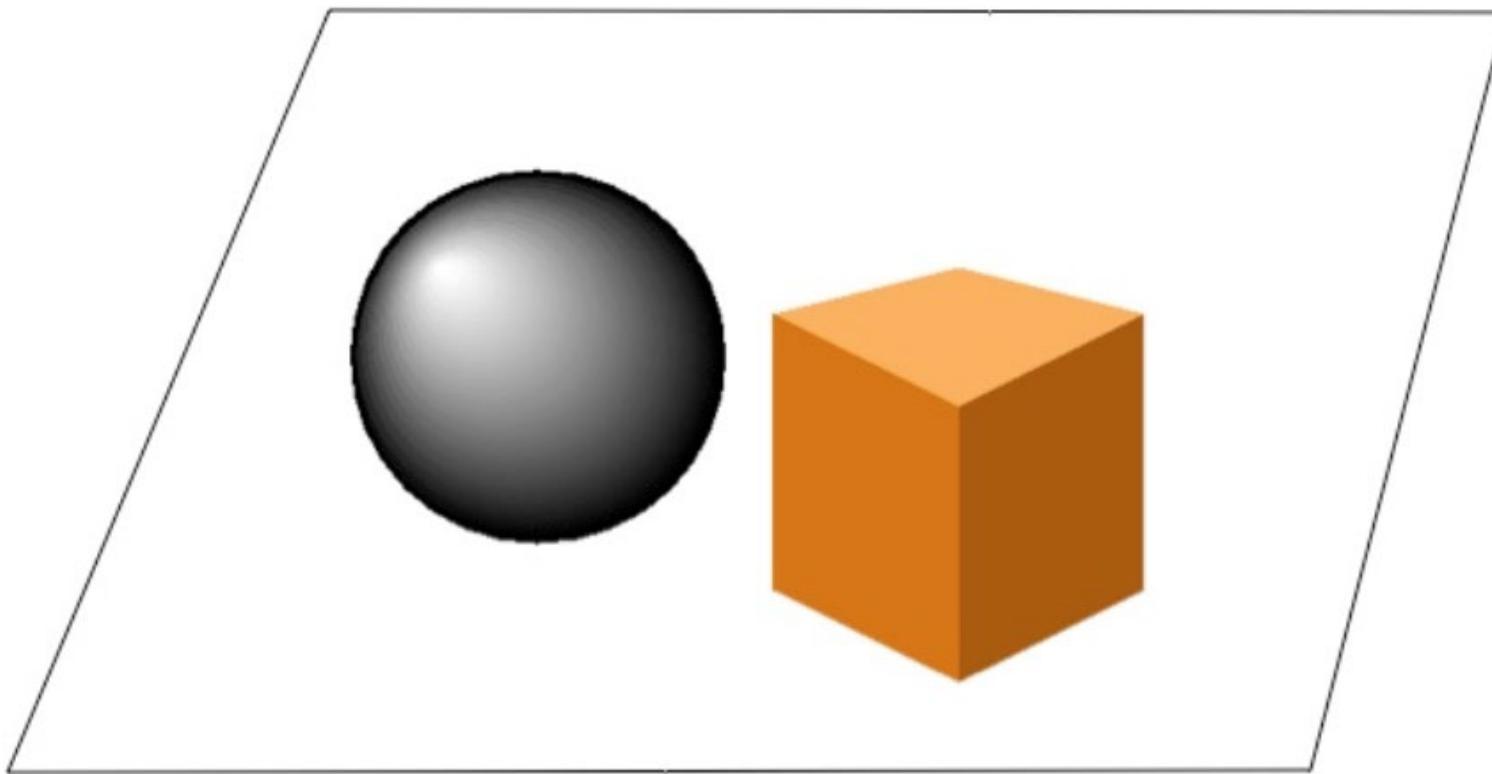
# Importance of shadows

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- Add realism to the scene.
- Shape, volume of the object.
- Position of light source.
- Depth perception.

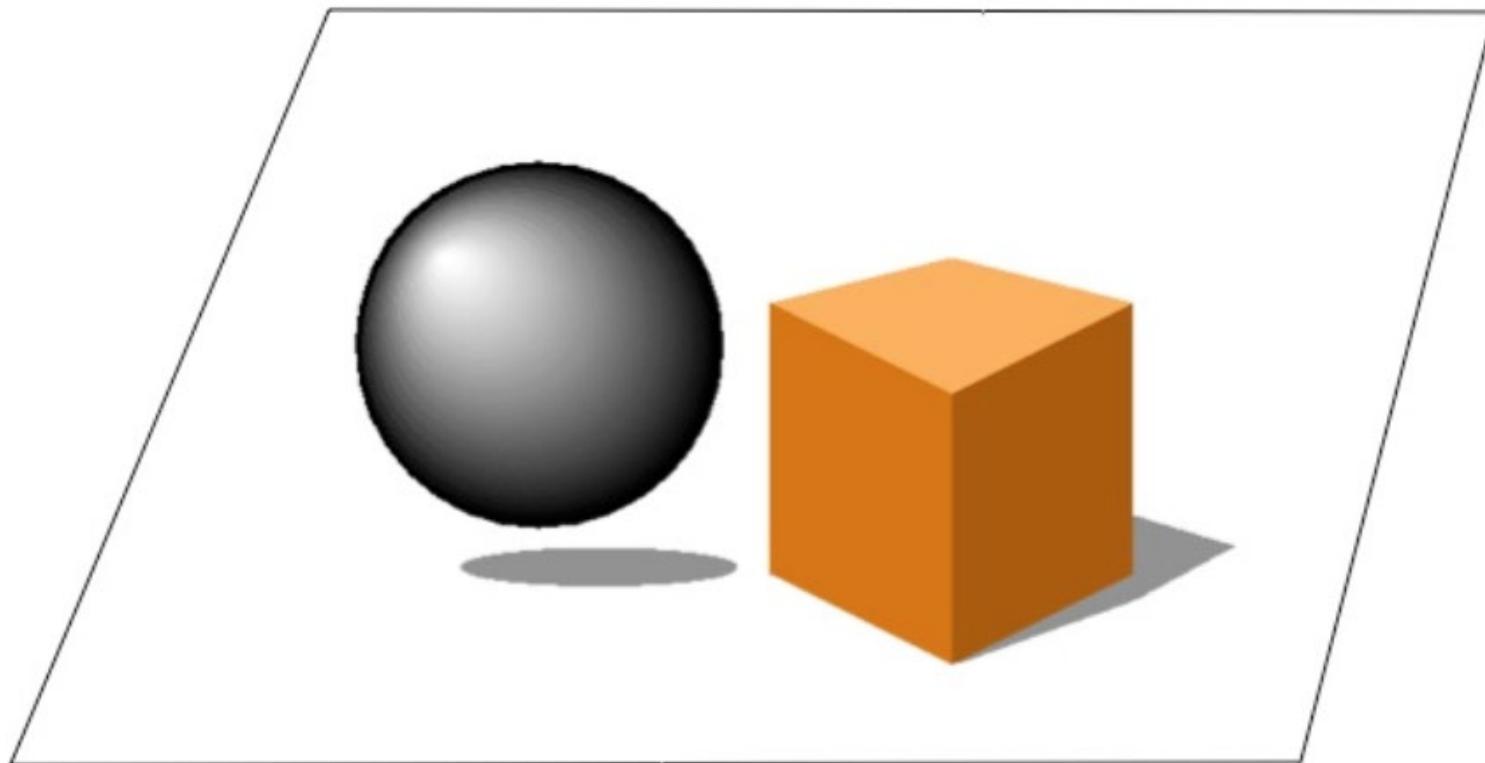
# Importance of shadows

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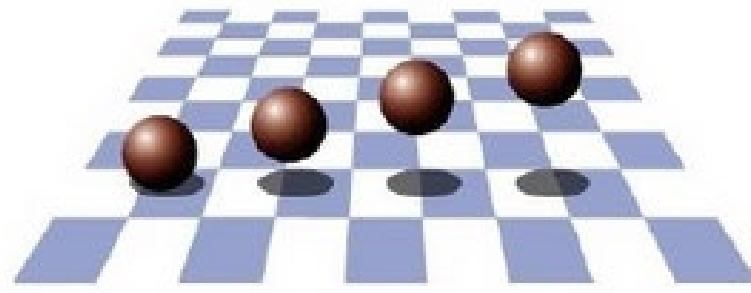
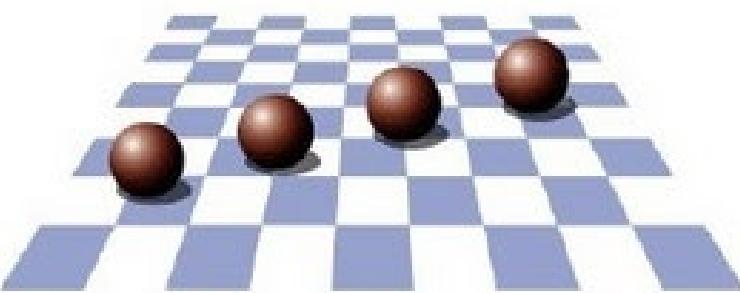
# Importance of shadows

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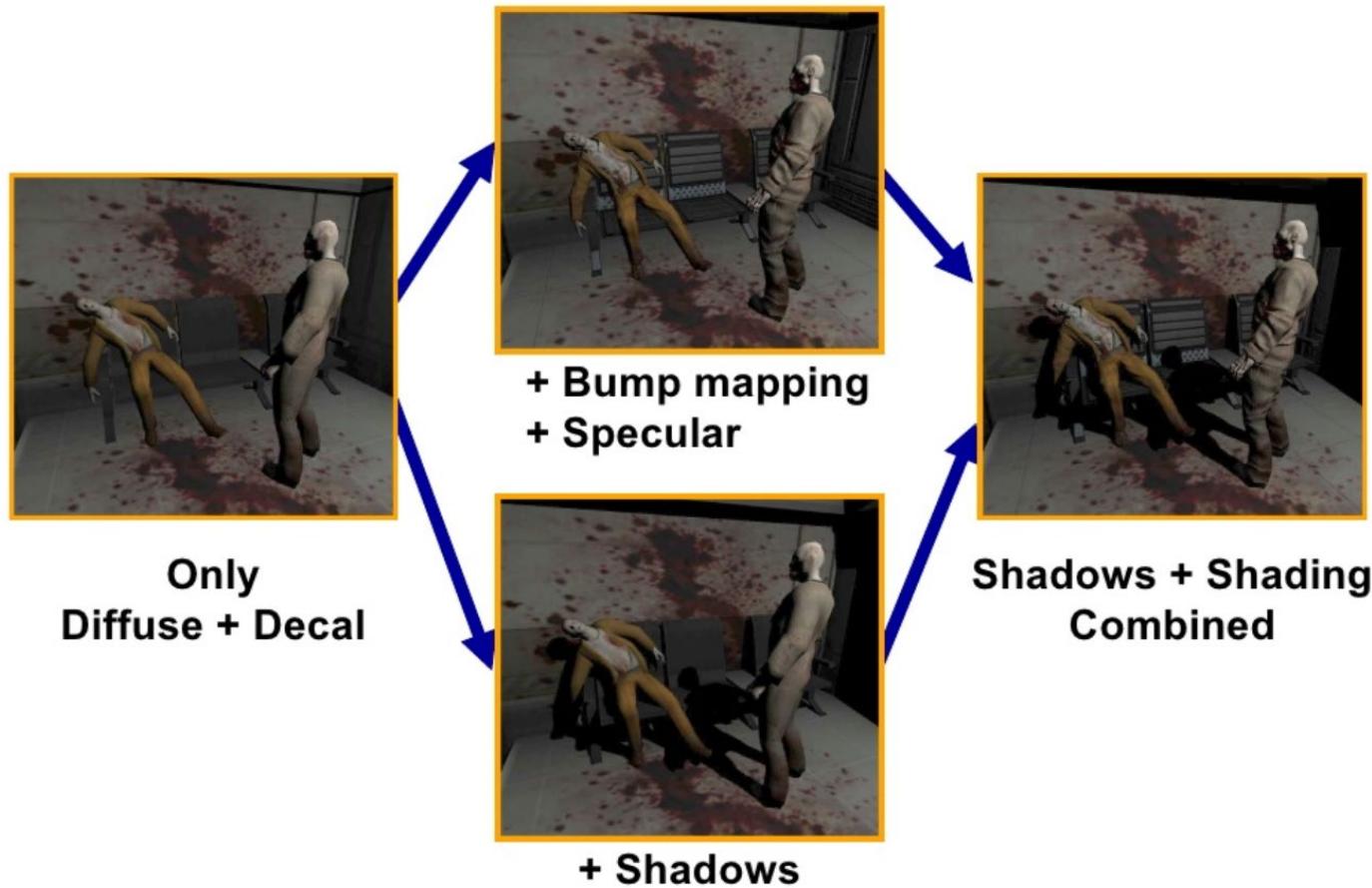


# Importance of shadows

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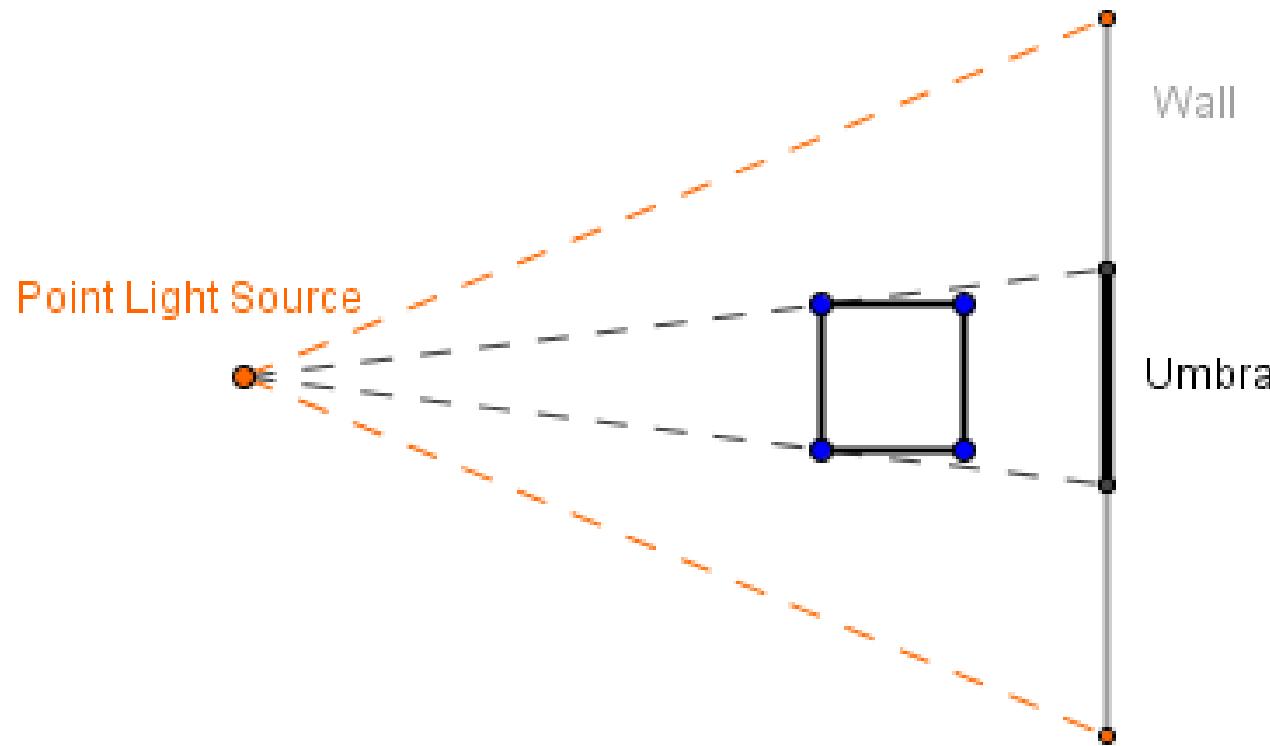


# Shading and shadows



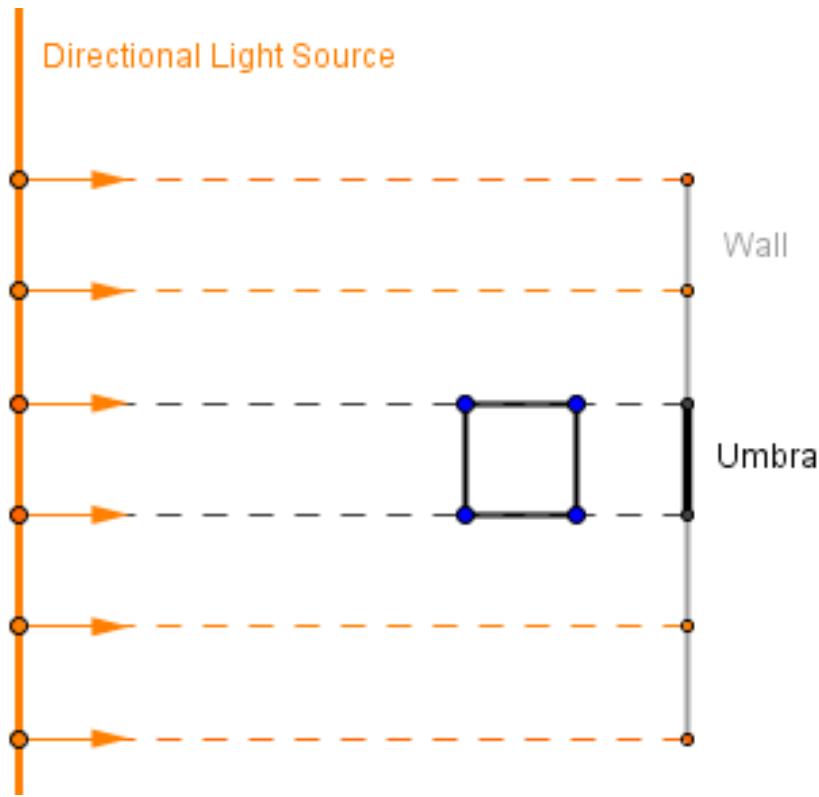
# Shadow components

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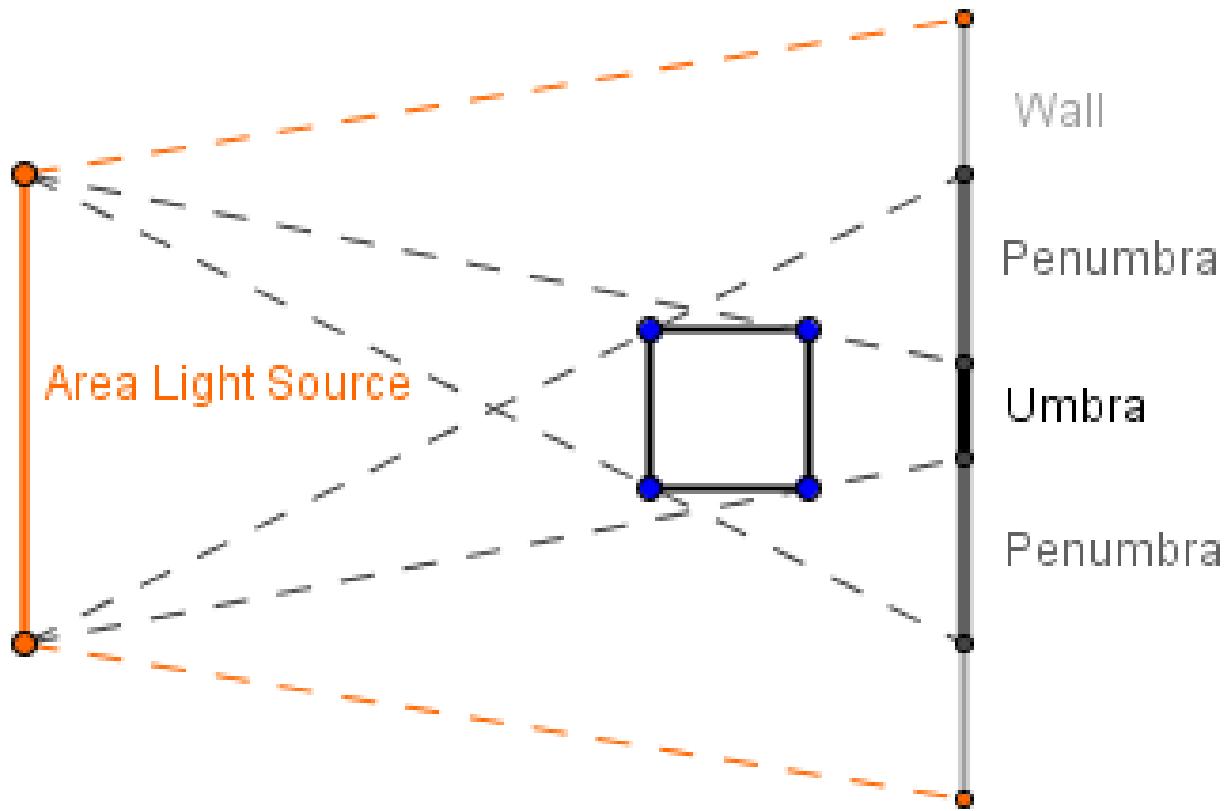


# Shadow components

---

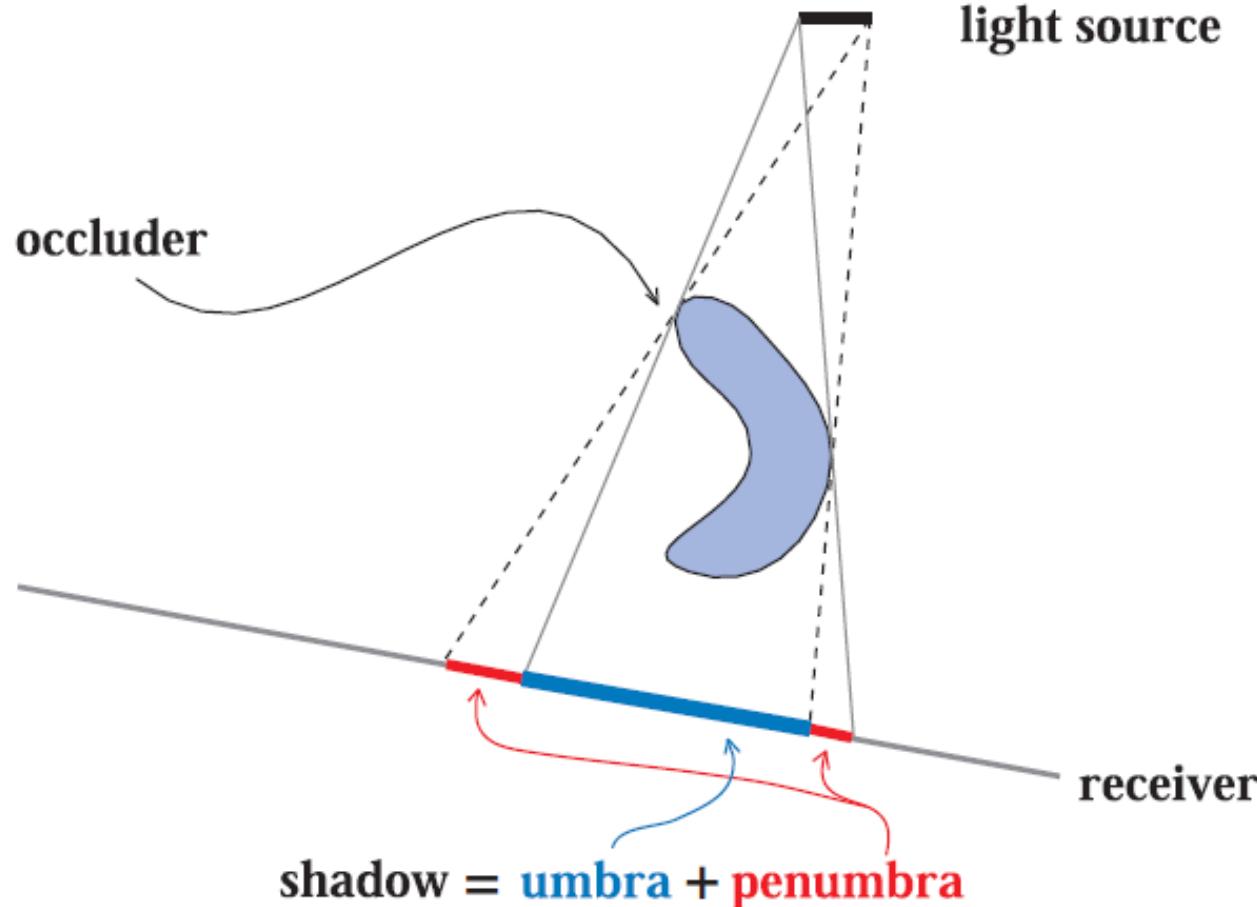


# Shadow components

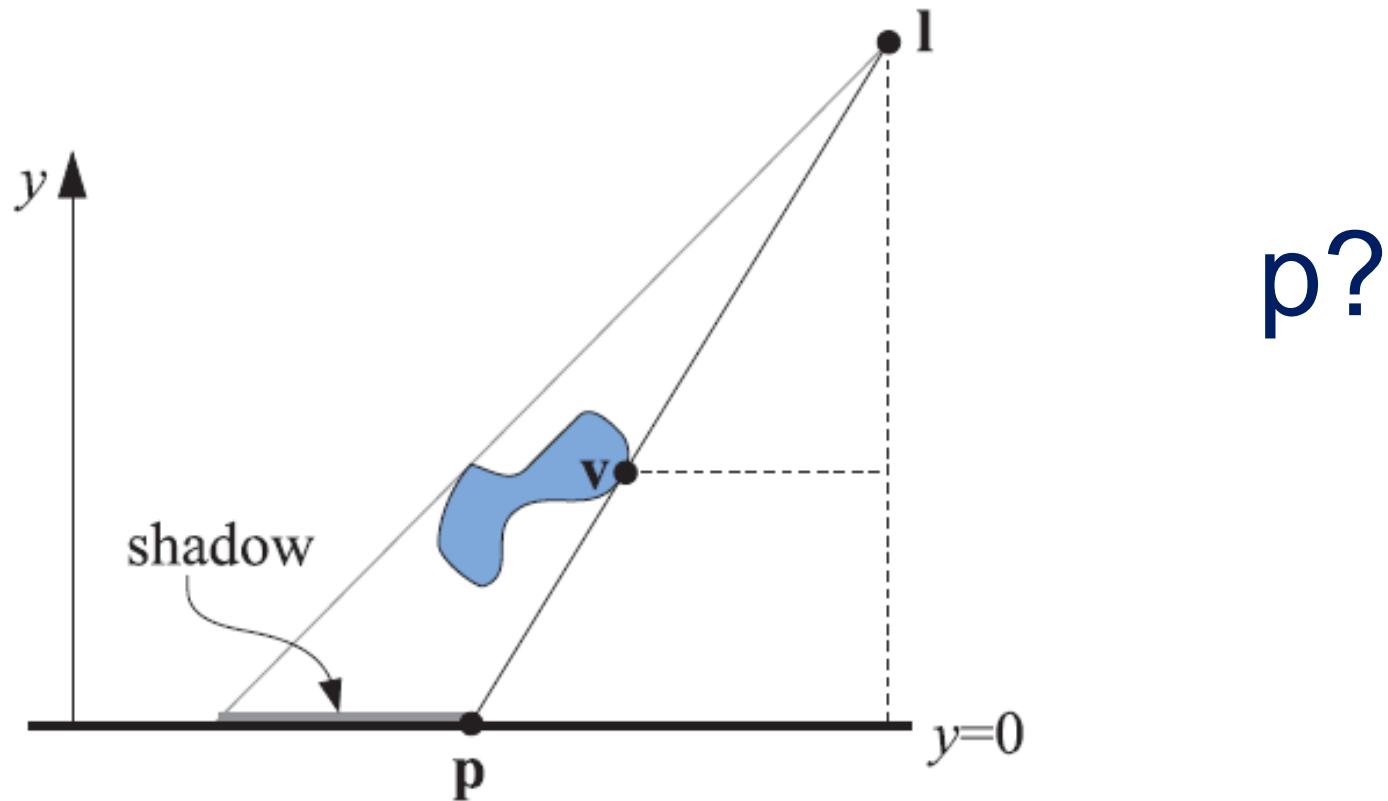


# Goal

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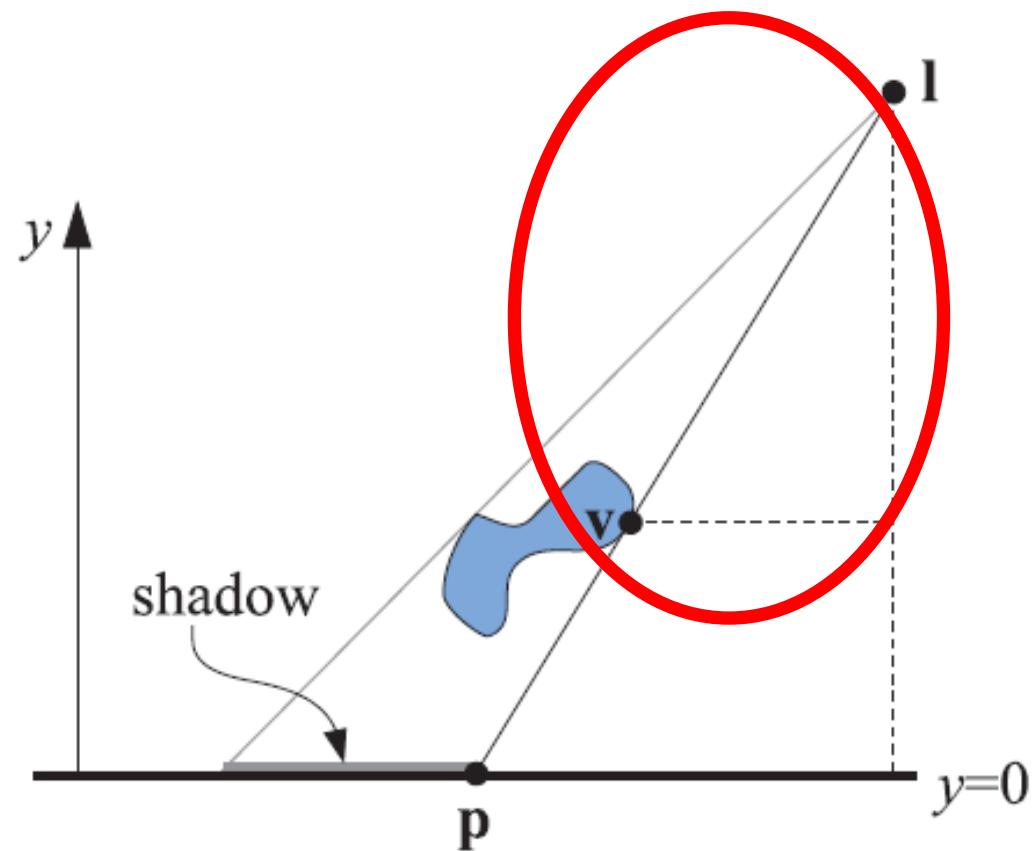


# Planar shadows

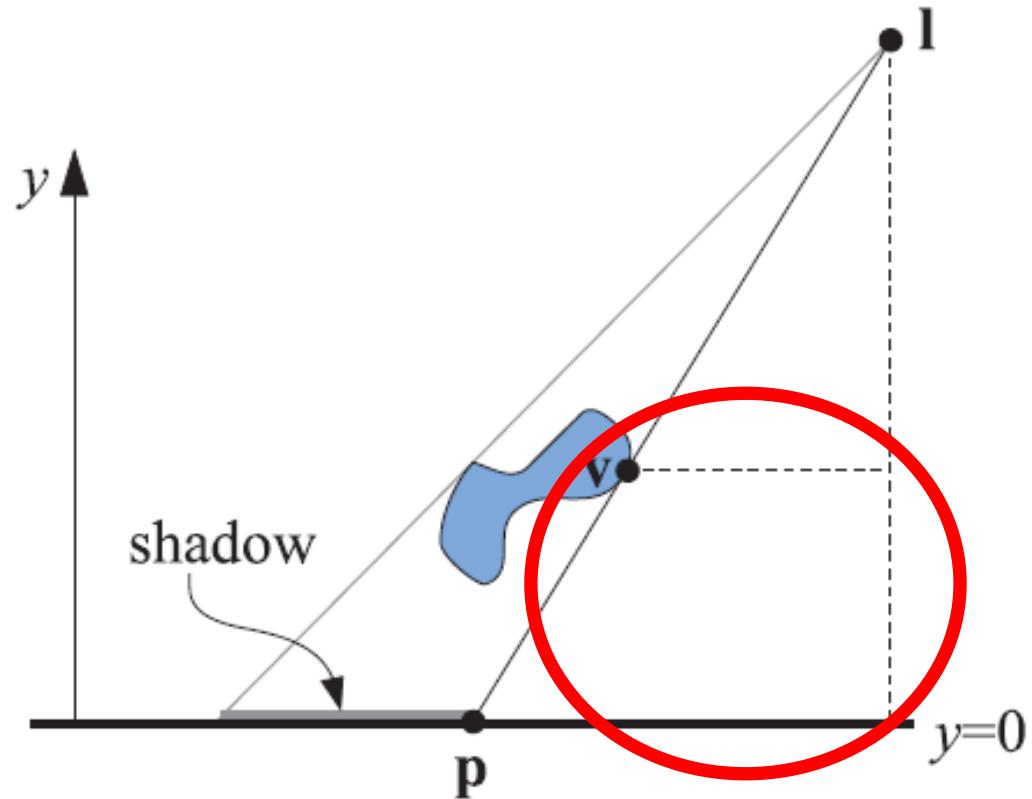


p?

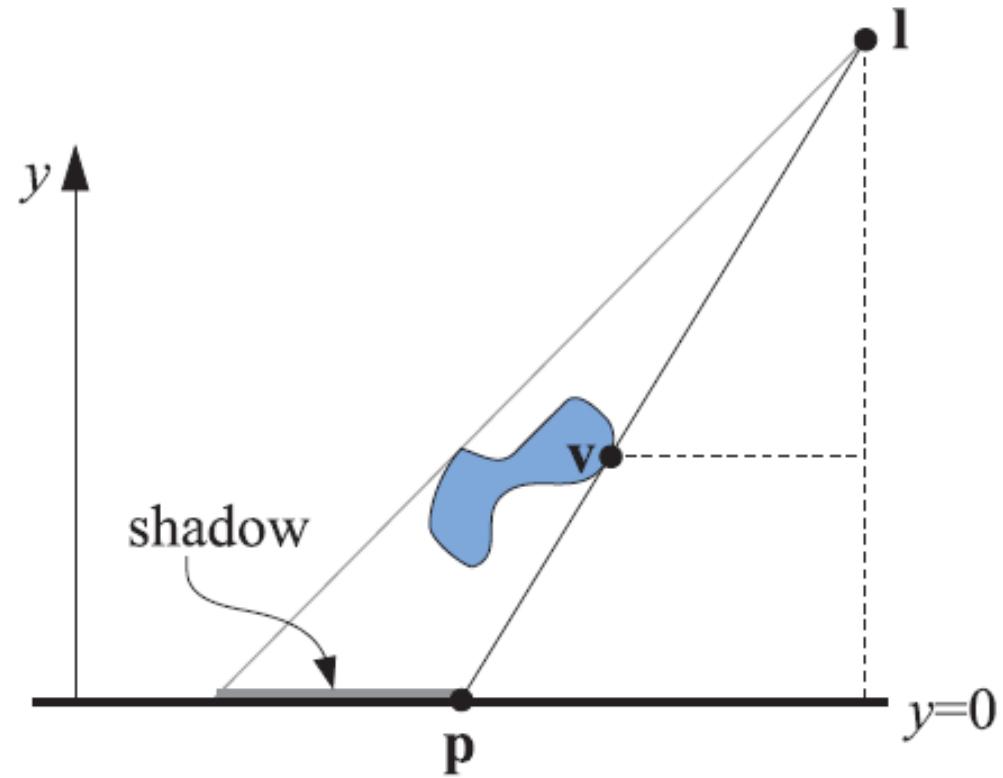
# Planar shadows



# Planar shadows



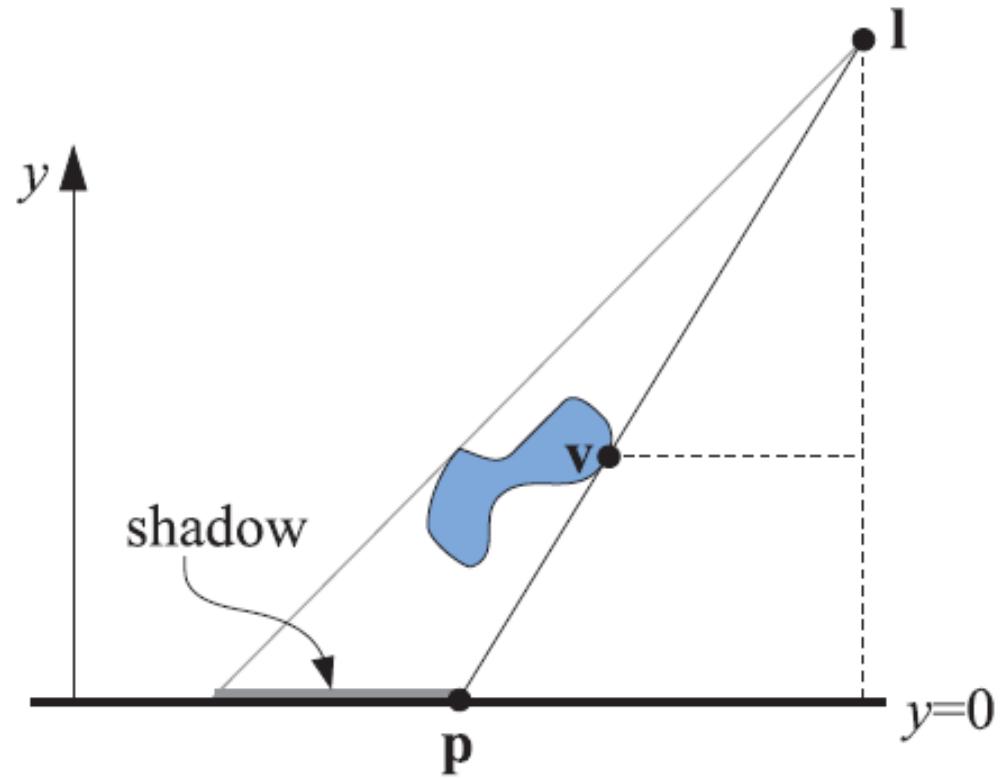
# Planar shadows



$$\frac{p_x - l_x}{v_x - l_x} = \frac{l_y}{l_y - v_y}$$

$$p_x = \frac{l_y v_x - l_x v_y}{l_y - v_y}$$

# Planar shadows



$$Mv = p$$

$$M = \begin{bmatrix} I_y & -I_x & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -I_z & I_y & 0 \\ 0 & -1 & 0 & I_y \end{bmatrix}$$

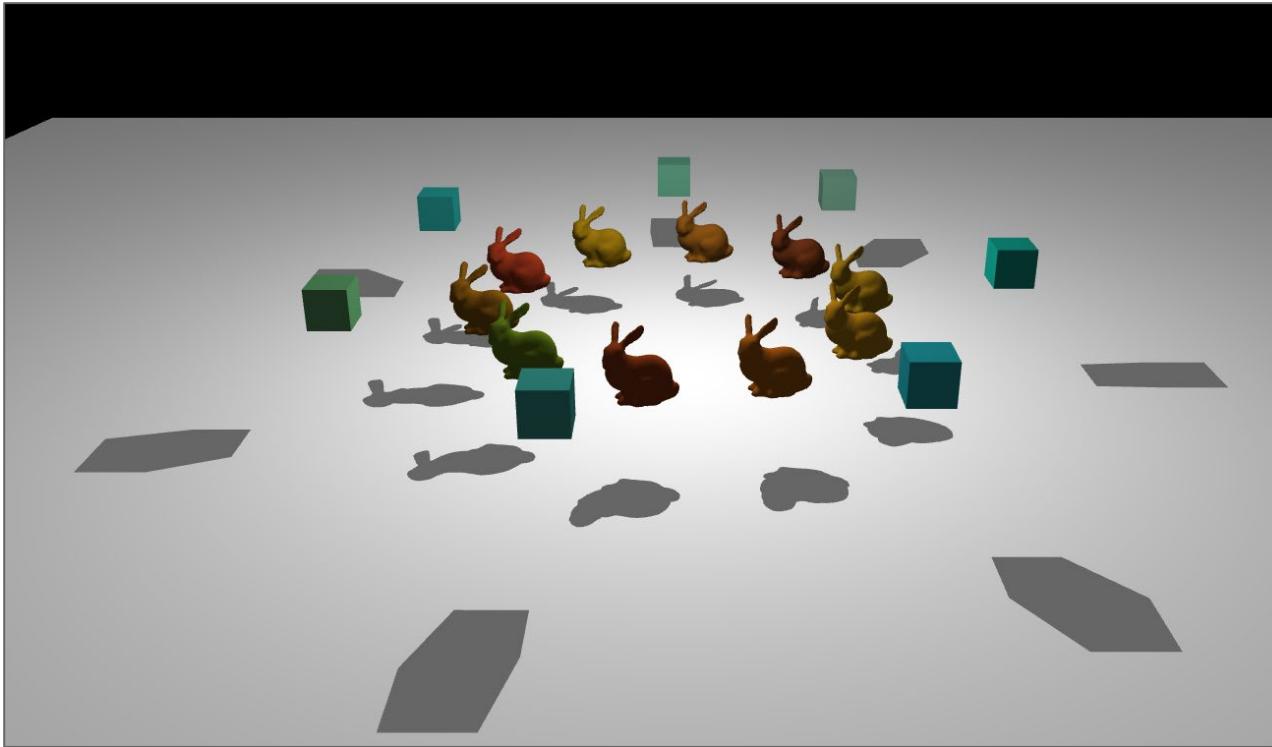
# Planar shadow: steps

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- Render receiving plane
- Render occluder, projecting with matrix M
- Render occluder

# Planar shadows: example

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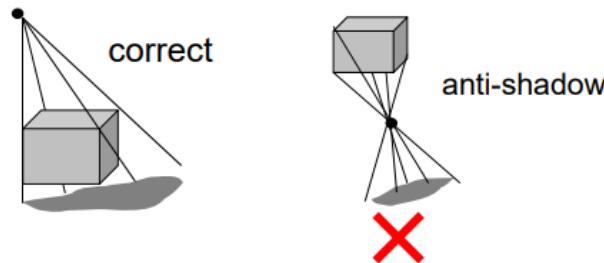


[https://erkaman.github.io/planar\\_proj\\_shadows/planar\\_proj\\_shadows.html](https://erkaman.github.io/planar_proj_shadows/planar_proj_shadows.html)

# Planar Shadows: shortcomings

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- Z-fighting:
  - Precision problem between receiving plane and occlude.
  - Use polygon offset (`glPolygonOffset`).
- Restricted to planar objects.
- Anti-shadows.



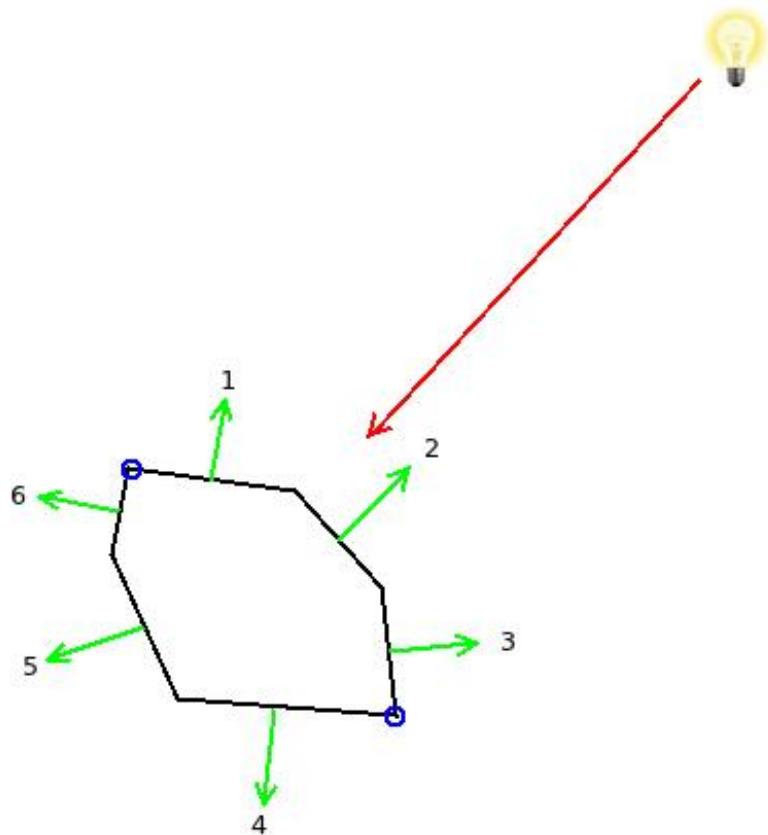
# Shadow volume

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# Silhouette detection

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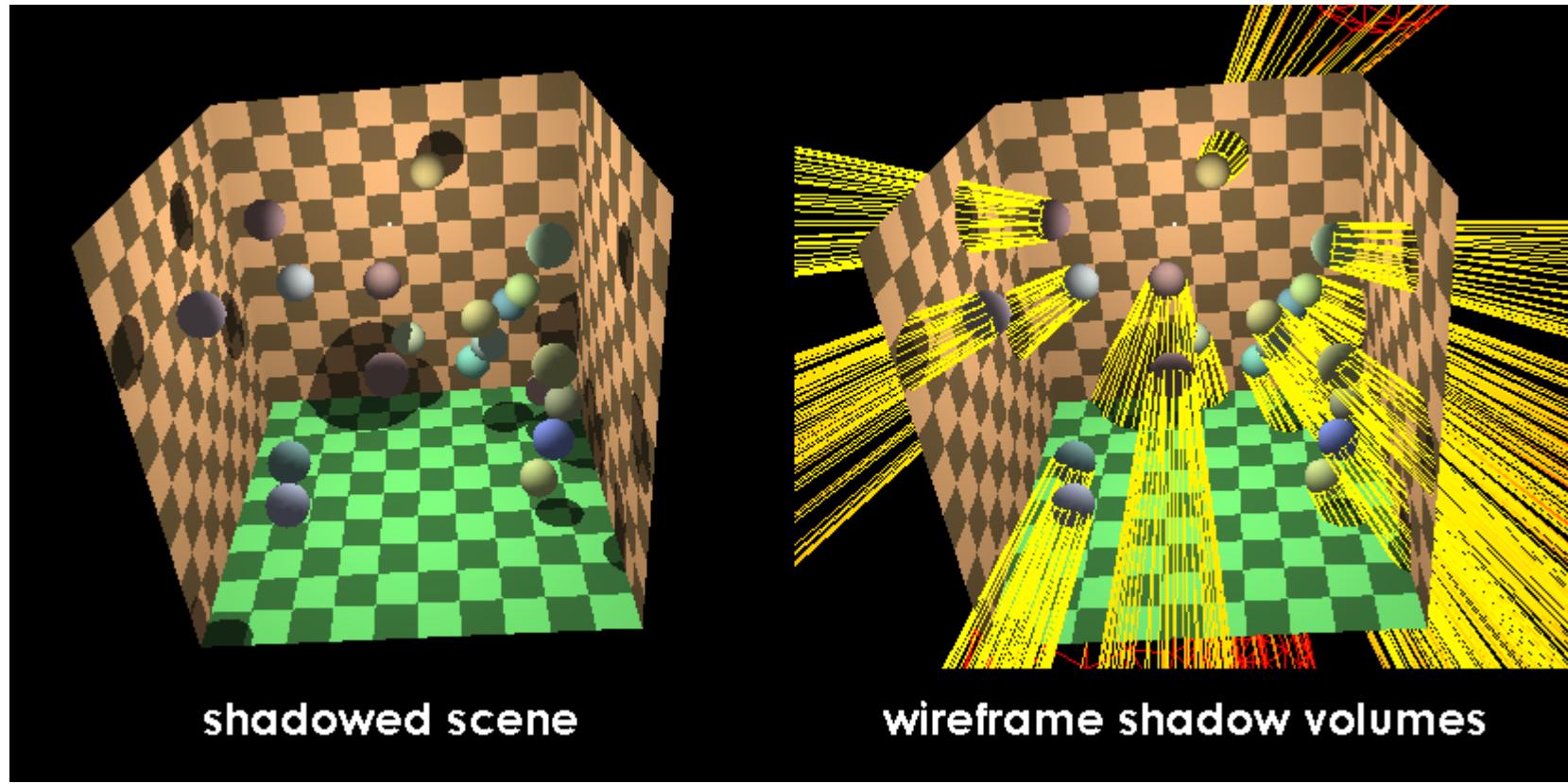


Silhouette:

- If one neighboring edge is facing the light, the other not.

# Silhouette detection

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# Silhouette detection

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How to find the edges?

Geometry shader

```
#version 330

layout (location = 0) in vec3 Position;
layout (location = 1) in vec2 TexCoord;
layout (location = 2) in vec3 Normal;

out vec3 WorldPos0;

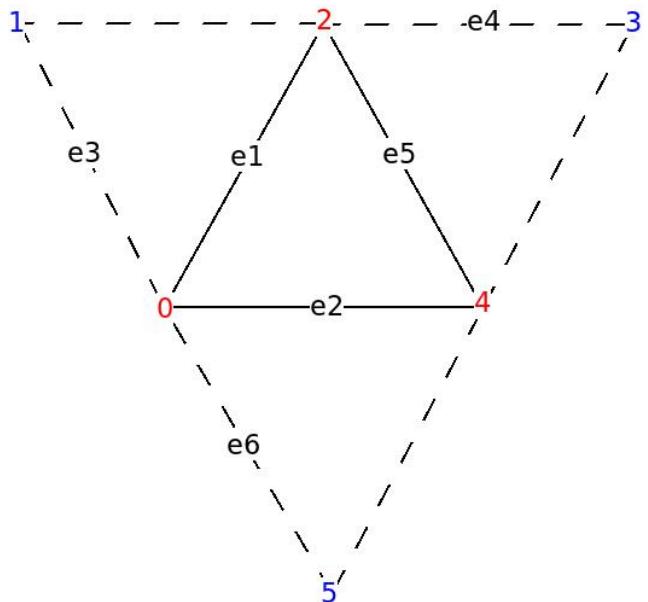
uniform mat4 gWVP;
uniform mat4 gWorld;

void main()
{
    vec4 PosL = vec4(Position, 1.0);
    gl_Position = gWVP * PosL;
    WorldPos0 = (gWorld * PosL).xyz;
}
```

Vertex Shader

# Silhouette detection

How to find the edges?  
Geometry shader



```
#version 330

layout (triangles_adjacency) in;
layout (line_strip, max_vertices = 6) out;

in vec3 WorldPos0[];
uniform vec3 gLightPos;

void main()
{
    vec3 e1 = WorldPos0[2] - WorldPos0[0];
    vec3 e2 = WorldPos0[4] - WorldPos0[0];
    vec3 e3 = WorldPos0[1] - WorldPos0[0];
    vec3 e4 = WorldPos0[3] - WorldPos0[2];
    vec3 e5 = WorldPos0[4] - WorldPos0[2];
    vec3 e6 = WorldPos0[5] - WorldPos0[0];

    vec3 Normal = cross(e1,e2);
    vec3 LightDir = gLightPos - WorldPos0[0];

    if (dot(Normal, LightDir) > 0.00001) {
        Normal = cross(e3,e1);

        if (dot(Normal, LightDir) <= 0) {
            // Silhouette!!!
        }

        Normal = cross(e4,e5);
        LightDir = gLightPos - WorldPos0[2];

        if (dot(Normal, LightDir) <= 0) {
            // Silhouette!!!
        }

        Normal = cross(e2,e6);
        LightDir = gLightPos - WorldPos0[4];

        if (dot(Normal, LightDir) <= 0) {
            // Silhouette!!!
        }
    }
}
```

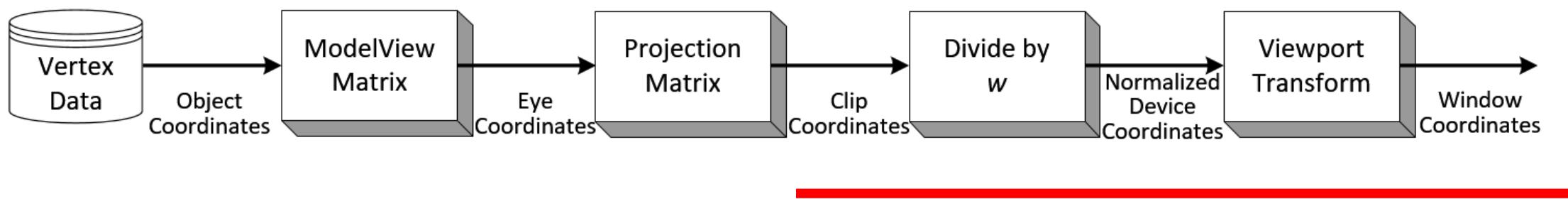
# Projection

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 4 \end{bmatrix}$$

$$\frac{(2,3,4)}{4} = (0.5, 0.75, 1)$$

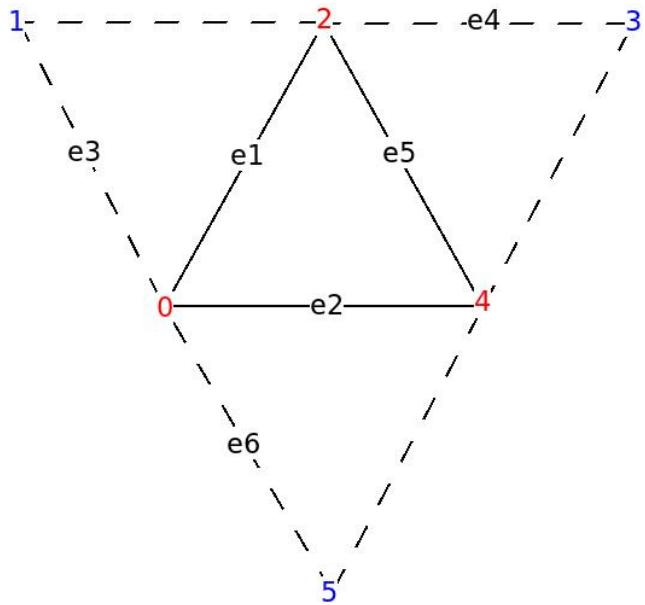
$$\frac{(2,3,4)}{0} = (\infty, \infty, \infty)$$

# Projection



# Silhouette detection

How to create volume?  
Geometry shader



```
// Emit a quad using a triangle strip
void EmitQuad(vec3 StartVertex, vec3 EndVertex)
{
    // Vertex #1: the starting vertex (just a tiny bit below the original edge)
    vec3 LightDir = normalize(StartVertex - gLightPos);
    gl_Position = gWVP * vec4((StartVertex + LightDir * EPSILON), 1.0);
    EmitVertex();

    // Vertex #2: the starting vertex projected to infinity
    gl_Position = gWVP * vec4(LightDir, 0.0);
    EmitVertex();

    // Vertex #3: the ending vertex (just a tiny bit below the original edge)
    LightDir = normalize(EndVertex - gLightPos);
    gl_Position = gWVP * vec4((EndVertex + LightDir * EPSILON), 1.0);
    EmitVertex();

    // Vertex #4: the ending vertex projected to infinity
    gl_Position = gWVP * vec4(LightDir , 0.0);
    EmitVertex();

    EndPrimitive();
}
```

# Projection

Far plane at infinity

$$P = \begin{pmatrix} \frac{1}{\text{aspectRatio} \cdot \tan(\frac{\alpha}{2})} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\frac{\alpha}{2})} & 0 & 0 \\ 0 & 0 & \frac{\text{near} + \text{far}}{\text{near} - \text{far}} & \frac{-2 \cdot \text{far} \cdot \text{near}}{\text{near} - \text{far}} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

# Projection

Far plane at infinity

$$\lim_{far \rightarrow \infty} -\frac{far + near}{far - near} = \lim_{far \rightarrow \infty} -\frac{\frac{far}{far} + \frac{near}{far}}{\frac{far}{far} - \frac{near}{far}} = -\frac{1 + 0}{1 - 0} = -1$$

$$P = \begin{pmatrix} \frac{1}{aspectRatio \cdot \tan(\frac{\alpha}{2})} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\frac{\alpha}{2})} & 0 & 0 \\ 0 & 0 & \frac{near+far}{near-far} & \frac{-2 \cdot far \cdot near}{near-far} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

# Projection

Far plane at infinity

$$P = \begin{pmatrix} \frac{1}{\text{aspectRatio} \cdot \tan(\frac{\alpha}{2})} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\frac{\alpha}{2})} & 0 & 0 \\ 0 & 0 & \frac{\text{near} + \text{far}}{\text{near} - \text{far}} & \frac{-2 \cdot \text{far} \cdot \text{near}}{\text{near} - \text{far}} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\lim_{\text{far} \rightarrow \infty} -\frac{\text{far} + \text{near}}{\text{far} - \text{near}} = \lim_{\text{far} \rightarrow \infty} -\frac{\frac{\text{far}}{\text{far}} + \frac{\text{near}}{\text{far}}}{\frac{\text{far}}{\text{far}} - \frac{\text{near}}{\text{far}}} = -\frac{1 + 0}{1 - 0} = -1$$

$$\lim_{\text{far} \rightarrow \infty} -\frac{2 \cdot \text{far} \cdot \text{near}}{\text{far} - \text{near}} = \lim_{\text{far} \rightarrow \infty} -\frac{\frac{2 \cdot \text{far} \cdot \text{near}}{\text{far}}}{\frac{\text{far}}{\text{far}} - \frac{\text{near}}{\text{far}}} = -\frac{2 \cdot \text{near}}{1 - 0} = -2 \cdot \text{near}$$

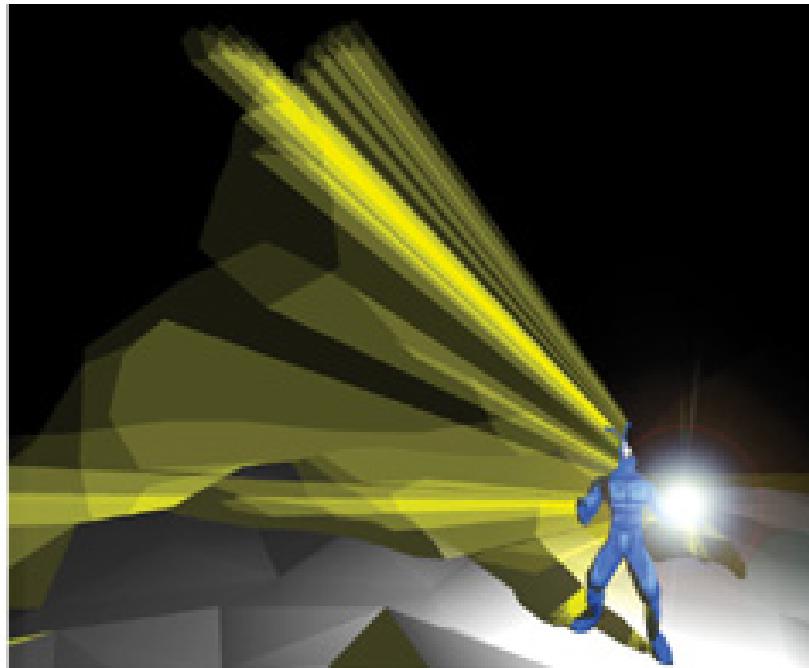
# Projection

Far plane at infinity

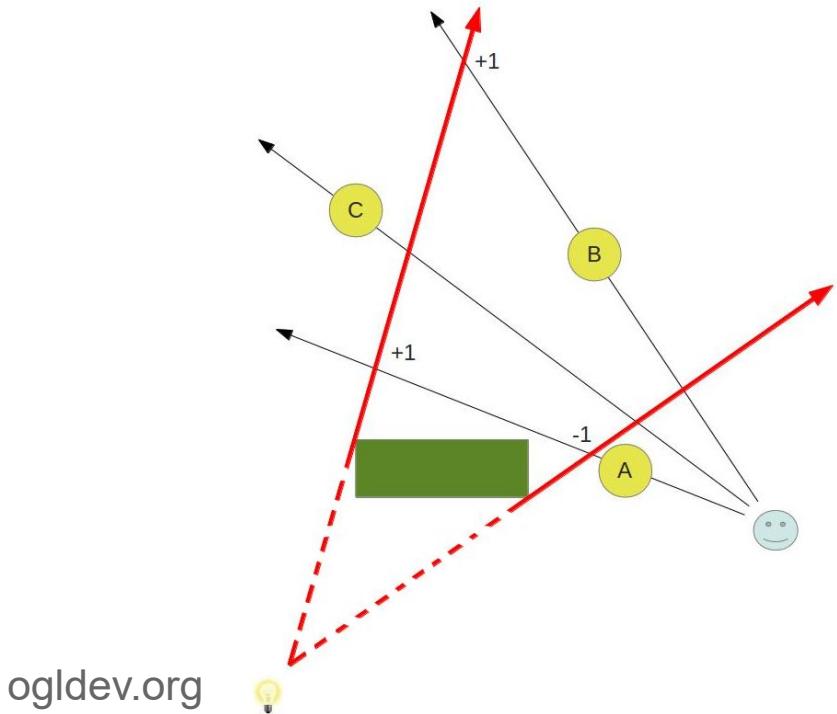
$$P_{\text{inf}} = \begin{pmatrix} \frac{1}{\text{aspectRatio} \cdot \tan(\frac{\alpha}{2})} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\frac{\alpha}{2})} & 0 & 0 \\ 0 & 0 & -1 & -2 \cdot \text{near} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

# Shadow volume

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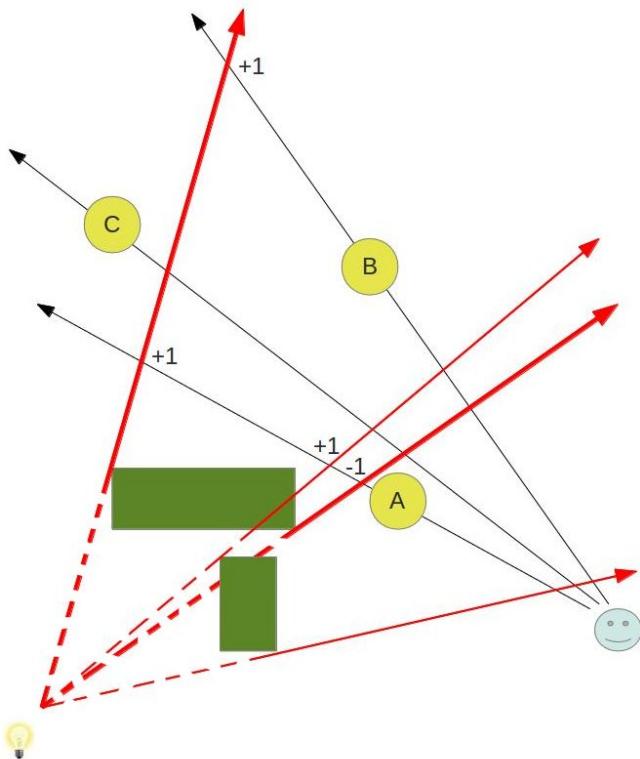
# Shadow volume



- Trace rays
  - Ray enters volume
    - Increase counter
  - Ray exists volume
    - Decreases counter
  - Shadow: counter different than zero

**Point  $P$  is in shadow if and only if there were more entering intersections than exiting intersections along a ray to infinity.**

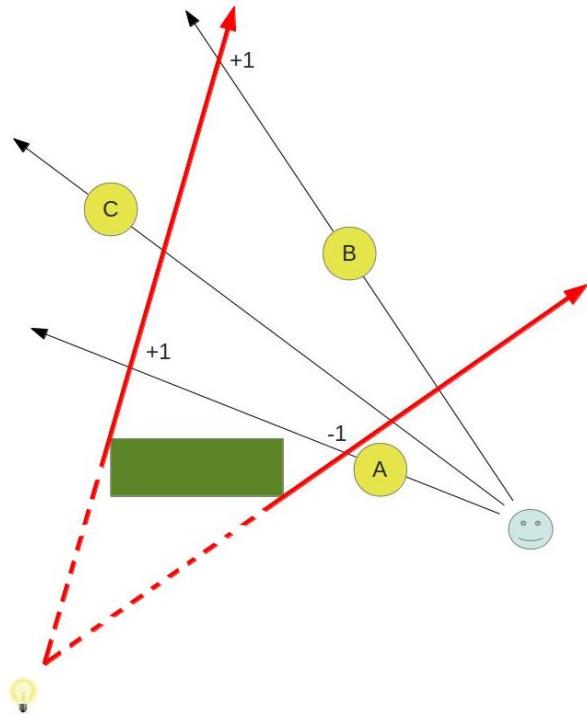
# Shadow volume



- Trace rays
  - Ray enters volume
    - Increase counter
  - Ray exists volume
    - Decreases counter
  - Shadow: counter different than zero

**Point  $P$  is in shadow if and only if there were more entering intersections than exiting intersections along a ray to infinity.**

# Shadow volume



- For each object, determine its shadow volume
- Render back facing polygons of volumes into stencil buffer
  - Depth test fail: increment
- Render front facing polygons of volume into stencil buffer
  - Depth test fail: decrement

**Point  $P$  is in shadow if and only if there were more entering intersections than exiting intersections along a ray to infinity.**

# Shadow volume

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- Advantages
  - Self-shadowing
  - Everything can shadow everything, including self
- Disadvantages
  - Silhouette computation required
  - Slow on scenes with polygons with large number of triangles

# Shadow maps

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- Image-space shadow determination.
- Leverages GPU hardware:
  - Depth buffering + texture mapping
- Two steps algorithm:
  - First, render scene from light's point of view.
  - Second, render scene from eye's point of view.

# Shadow maps

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- First step:
  - Render from light's point of view:
    - Result stored in a depth buffer, as a shadow map.
    - A 2D function indicating the depth of the closest pixel to the light.
  - Shadow map is used in the second step.

# Shadow maps

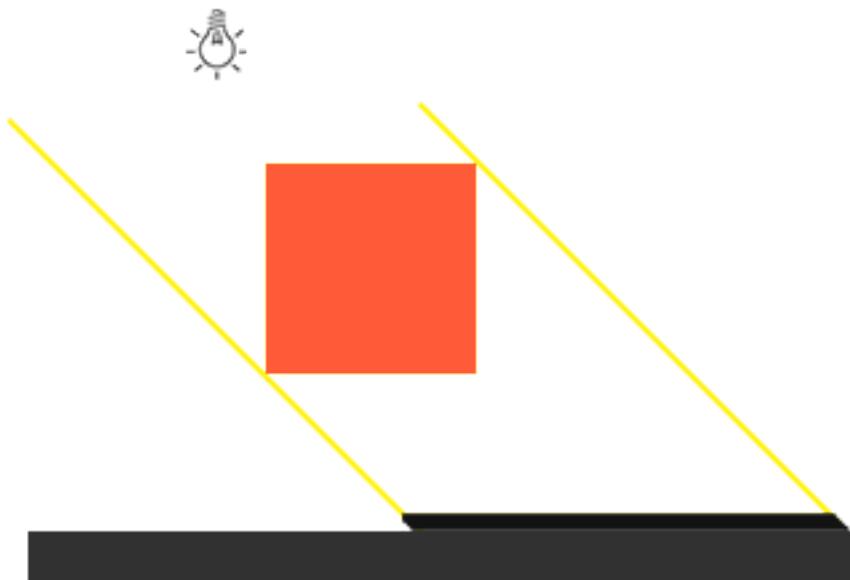
---

- Second step:
  - Render from eye's point of view:
    - For each fragment, determine its position in the light space.
    - Compare depth value at light position with the depth value from shadow map.
  - Two values:
    - A: z value of fragment in light space.
    - B: z value of fragment in shadow map.
    - $B > A$ : shadow
    - $A > B$ : no shadow

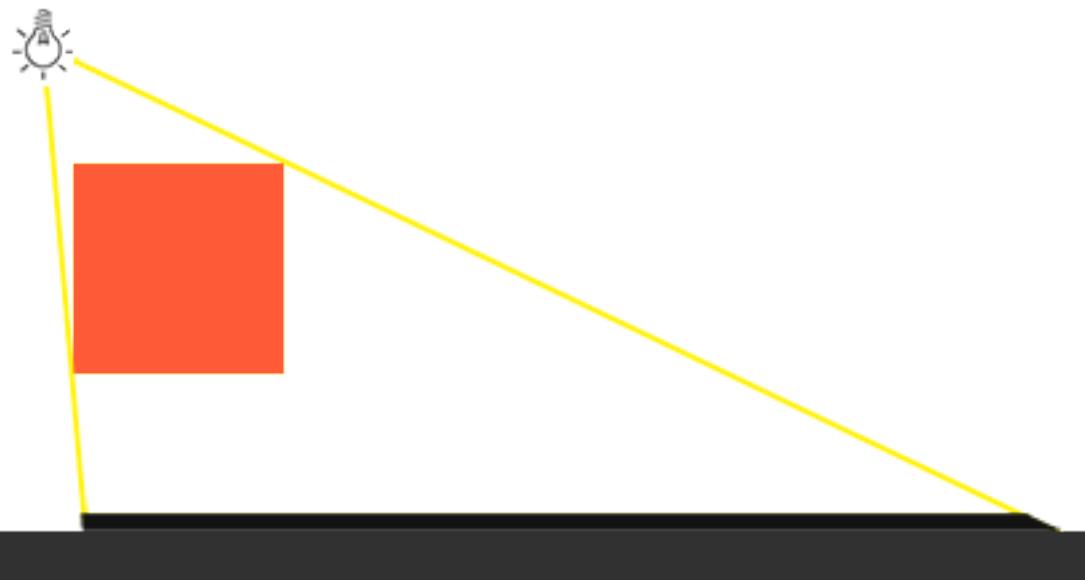
# Shadow maps

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ORTHOGRAPHIC PROJECTION

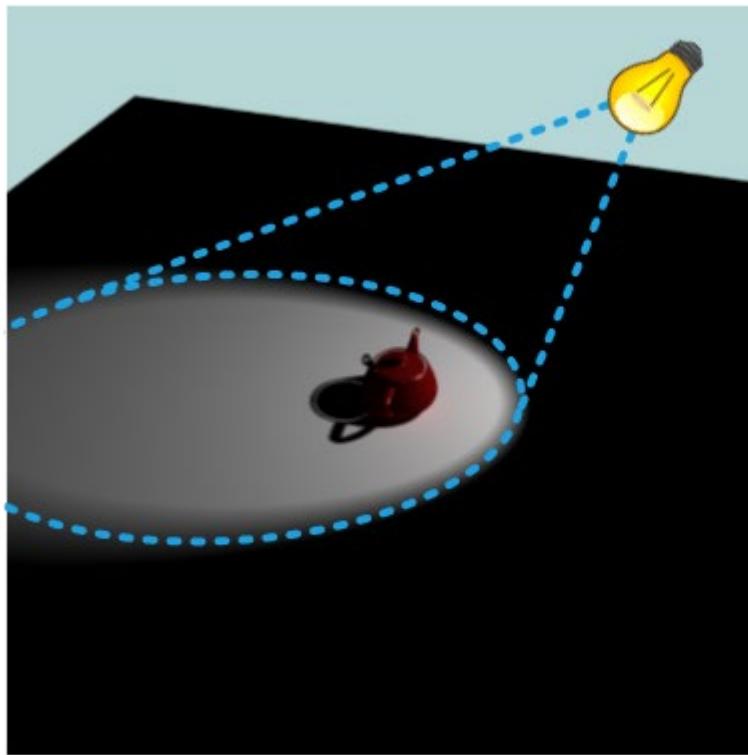


PERSPECTIVE PROJECTION

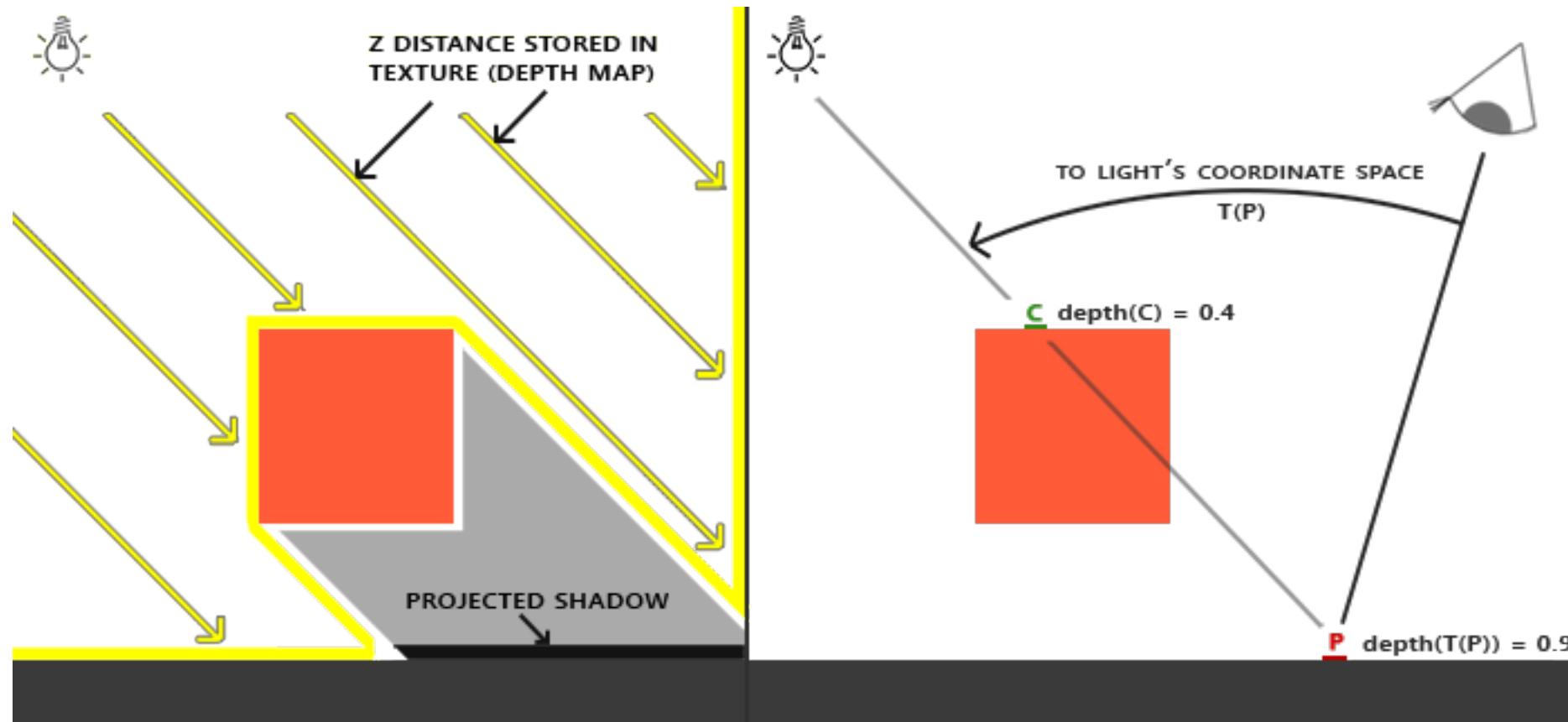


# Shadow test

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# Shadow test



# Shadow maps + GL

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1. Creating shadow map framebuffer for rendering shadow map:

```
unsigned int depthMapFBO;  
glGenFramebuffers(1, &depthMapFBO);
```

2. Creating shadow map texture with size 1024

```
const unsigned int SHADOW_WIDTH = 1024, SHADOW_HEIGHT = 1024;  
  
unsigned int depthMap;  
glGenTextures(1, &depthMap);  
 glBindTexture(GL_TEXTURE_2D, depthMap);  
 glTexImage2D(GL_TEXTURE_2D, 0, GL_DEPTH_COMPONENT,  
             SHADOW_WIDTH, SHADOW_HEIGHT, 0, GL_DEPTH_COMPONENT, GL_FLOAT, NULL);  
 glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MIN_FILTER, GL_NEAREST);  
 glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MAG_FILTER, GL_NEAREST);  
 glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_WRAP_S, GL_REPEAT);  
 glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_WRAP_T, GL_REPEAT);
```

Snippets from: <https://learnopengl.com/>

# Shadow maps + GL

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## 3. Attach texture to framebuffer

```
unsigned int depthMapFBO;  
glGenFramebuffers(1, &depthMapFBO);
```

```
glBindFramebuffer(GL_FRAMEBUFFER, depthMapFBO);  
glFramebufferTexture2D(GL_FRAMEBUFFER, GL_DEPTH_ATTACHMENT, GL_TEXTURE_  
2D, depthMap, 0);  
glDrawBuffer(GL_NONE);  
glReadBuffer(GL_NONE);  
glBindFramebuffer(GL_FRAMEBUFFER, 0);
```

# Shadow maps + GL

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## 4. Render scene twice

```
// 1. first render to depth map
glViewport(0, 0, SHADOW_WIDTH, SHADOW_HEIGHT);
glBindFramebuffer(GL_FRAMEBUFFER, depthMapFBO);
    glClear(GL_DEPTH_BUFFER_BIT);
    ConfigureShaderAndMatrices();
    RenderScene();
glBindFramebuffer(GL_FRAMEBUFFER, 0);
// 2. then render scene as normal with shadow mapping (using depth map)
glViewport(0, 0, SCR_WIDTH, SCR_HEIGHT);
glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT);
ConfigureShaderAndMatrices();
glBindTexture(GL_TEXTURE_2D, depthMap);
RenderScene();
```

# Shadow maps + GL

## 4. Shadow test: vertex shader

```
#version 330 core
layout (location = 0) in vec3 aPos;
layout (location = 1) in vec3 aNormal;
layout (location = 2) in vec2 aTexCoords;

out VS_OUT {
    vec3 FragPos;
    vec3 Normal;
    vec2 TexCoords;
    vec4 FragPosLightSpace;
} vs_out;

uniform mat4 projection;
uniform mat4 view;
uniform mat4 model;
uniform mat4 lightSpaceMatrix;

void main()
{
    vs_out.FragPos = vec3(model * vec4(aPos, 1.0));
    vs_out.Normal = transpose(inverse(mat3(model))) * aNormal;
    vs_out.TexCoords = aTexCoords;
    vs_out.FragPosLightSpace = lightSpaceMatrix * vec4(vs_out.FragPos, 1.0);
    gl_Position = projection * view * vec4(vs_out.FragPos, 1.0);
}
```

# Shadow maps + GL

## 4. Shadow test: fragment shader

```
float ShadowCalculation(vec4 fragPosLightSpace)
{
    // perform perspective divide
    vec3 projCoords = fragPosLightSpace.xyz / fragPosLightSpace.w;
    [...]
}
```

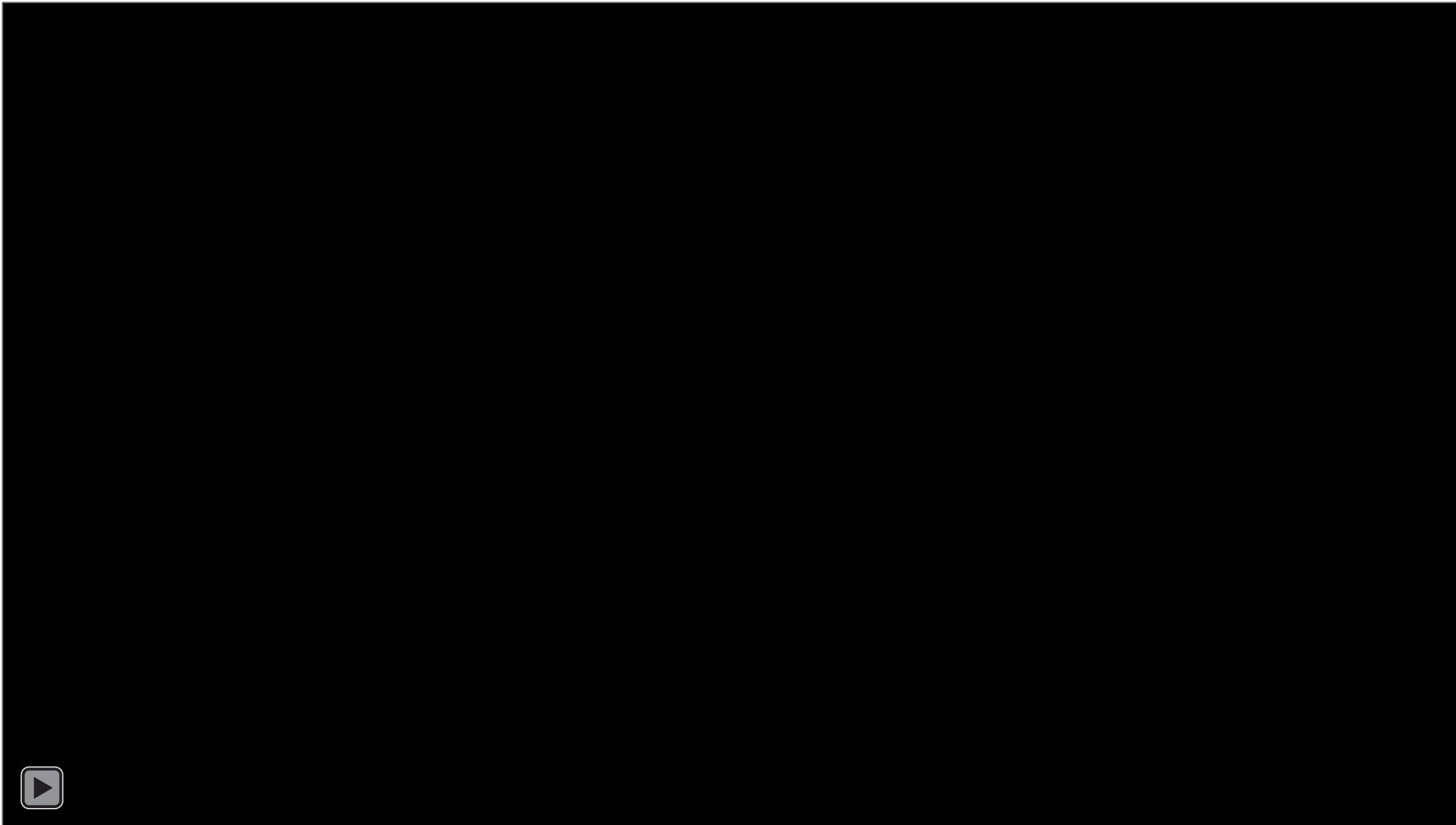
```
projCoords = projCoords * 0.5 + 0.5;
```

```
float closestDepth = texture(shadowMap, projCoords.xy).r;
```

```
float currentDepth = projCoords.z;
float shadow = currentDepth > closestDepth ? 1.0 : 0.0;
```

# Shadow map result

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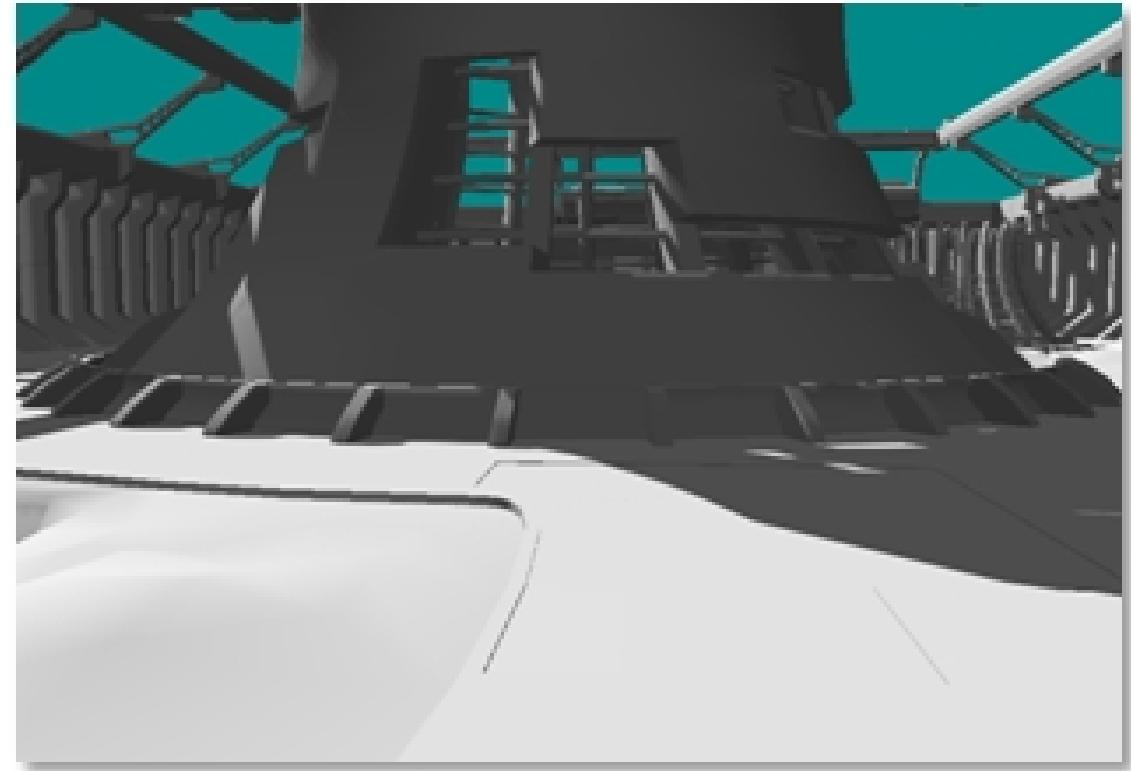
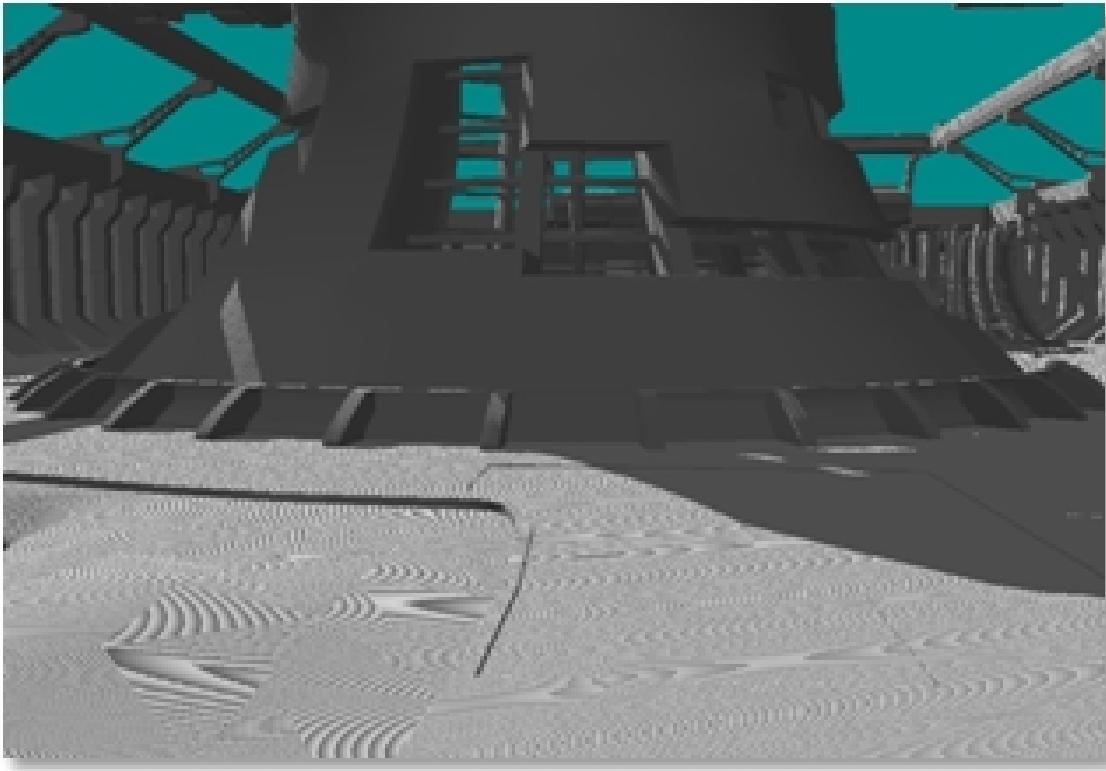
# Shadow maps

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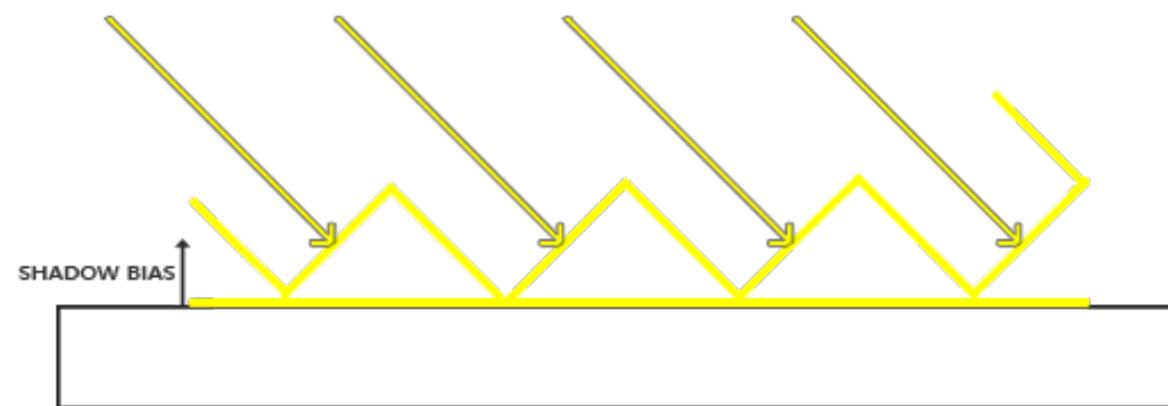
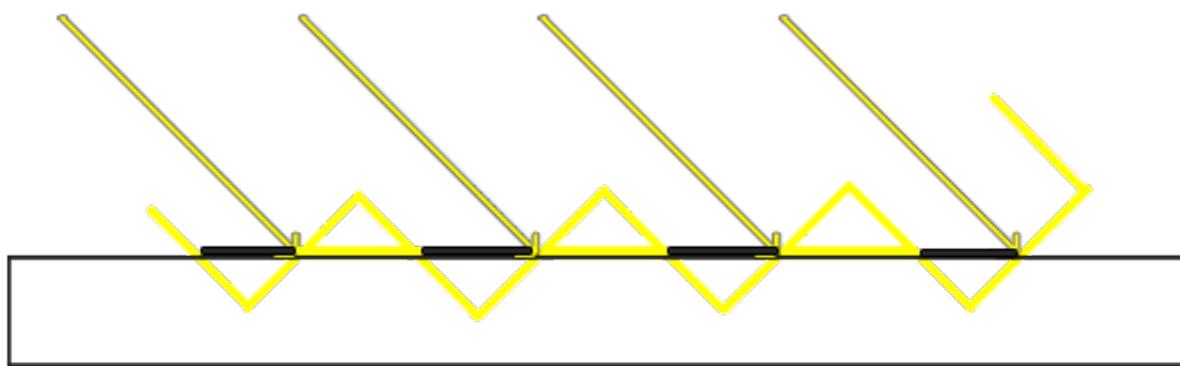
- Advantages:
  - Fast
  - Simple depth map comparison

# Shadow acne

---

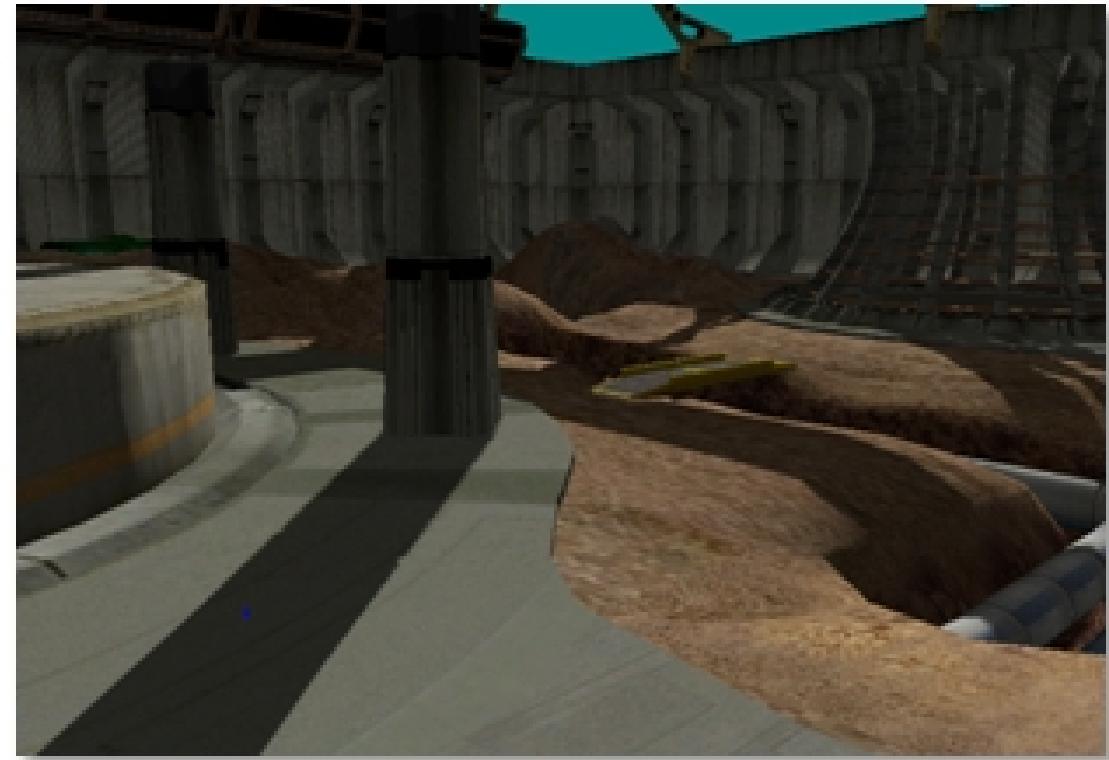


# Shadow acne



# Peter panning

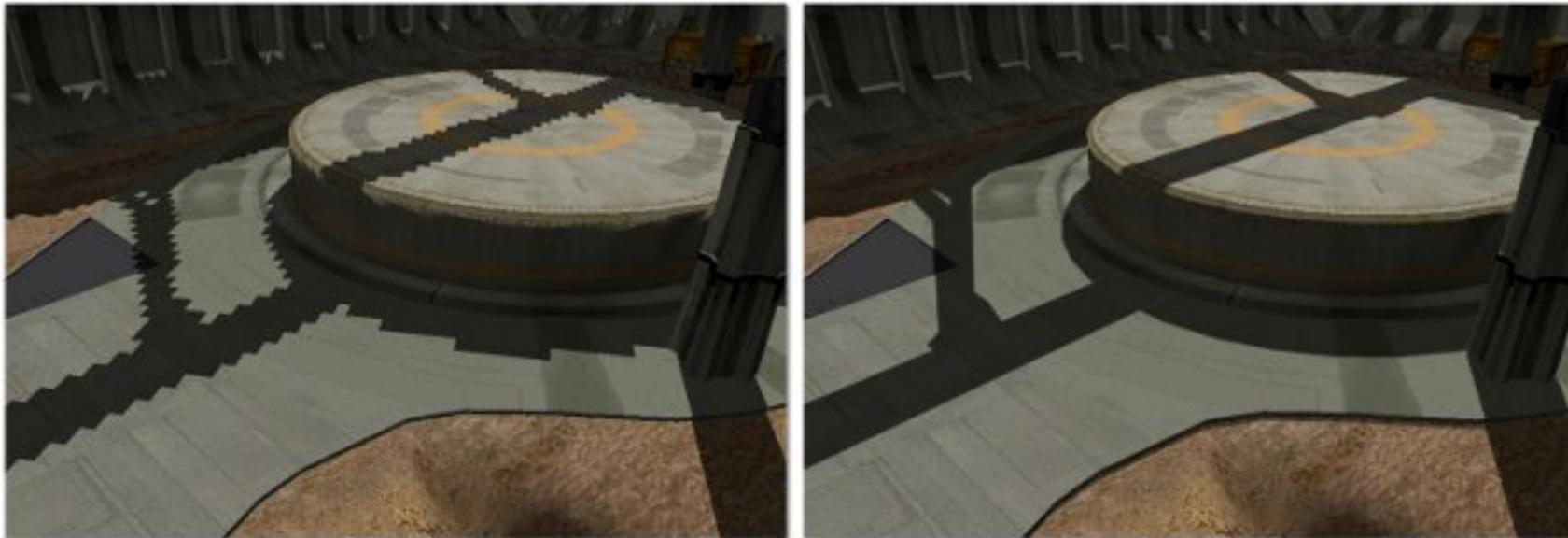
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# Shadow aliasing

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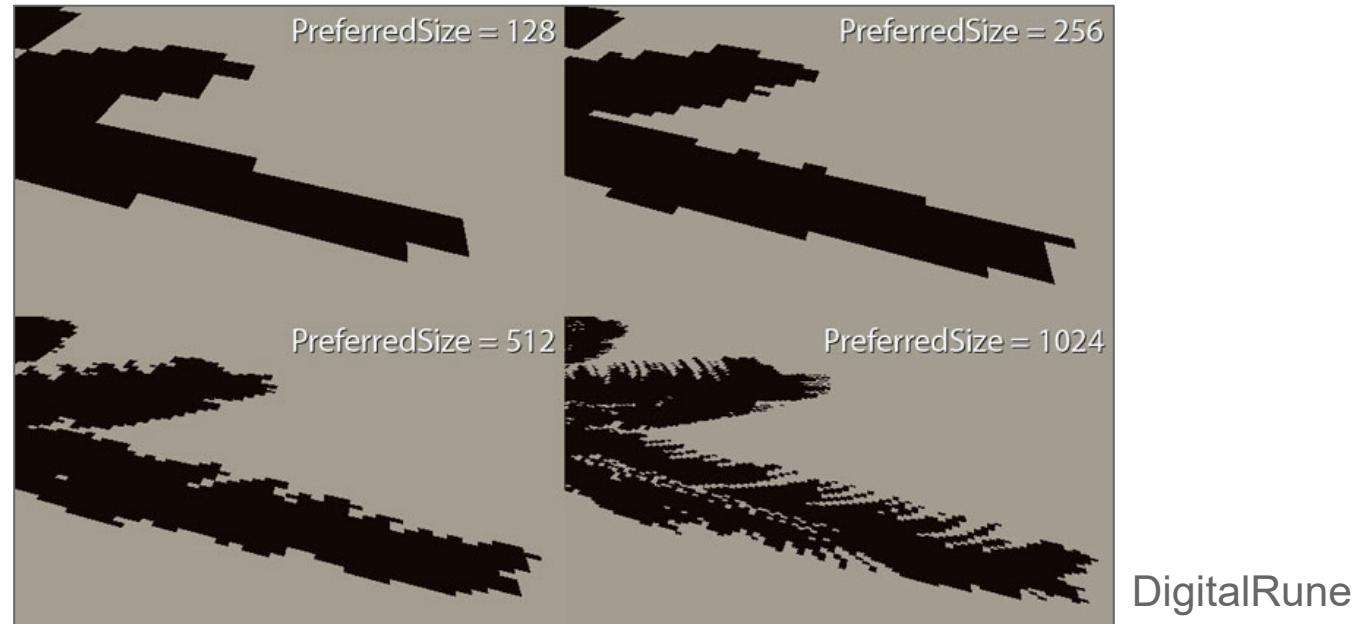
- Finite shadow map resolution: pixelized shadows.
- Large scenes require high shadow map resolution, and a tight projection.



# Shadow aliasing

---

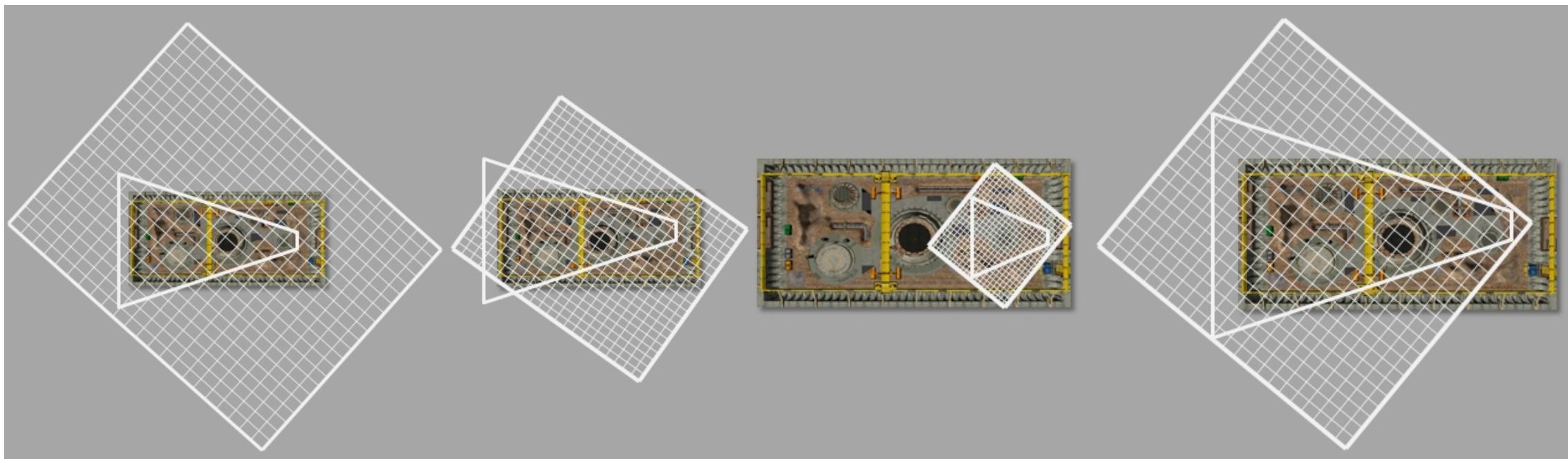
- Finite shadow map resolution: pixelized shadows.
- Large scenes require high shadow map resolution, and a tight projection.



# Shadow aliasing

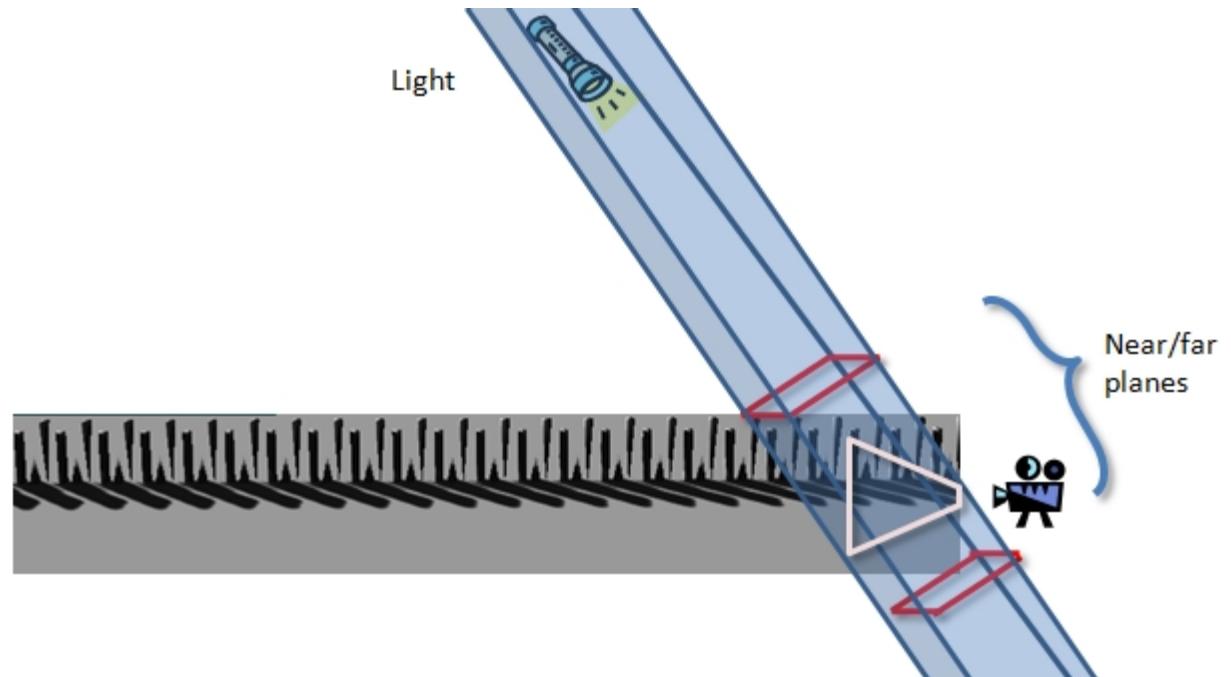
---

- Calculate a tight projection. How?



# Computing optimal projection

- Calculate 8 corners of view frustum in light space.
- 6 planes: 4 sides, near, far.
- Clip scene's bounds against 4 side planes.
- Smallest and largest z-values of clipped boundaries represent the near and far plan, respectively.



# Shadow aliasing

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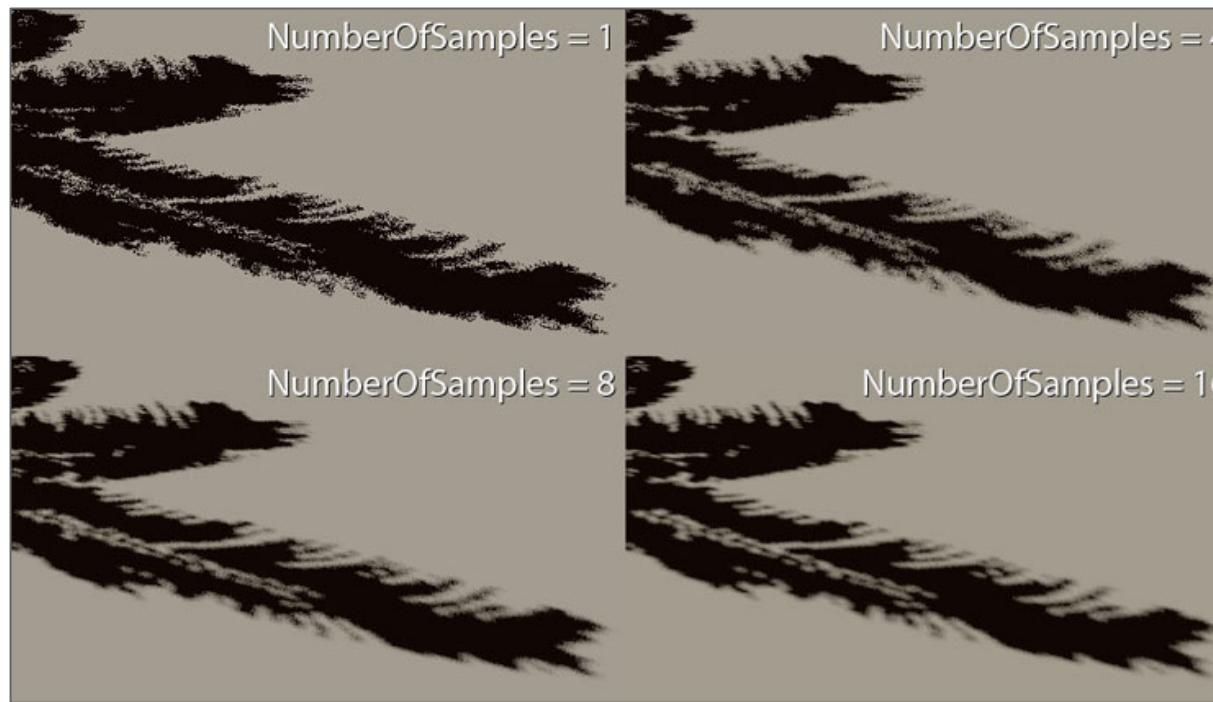
- Solution: sample from more than once from shadow map

```
float shadow = 0.0;
vec2 texelSize = 1.0 / textureSize(shadowMap, 0);
for(int x = -1; x <= 1; ++x)
{
    for(int y = -1; y <= 1; ++y)
    {
        float pcfDepth = texture(shadowMap, projCoords.xy + vec2(x, y)
* texelSize).r;
        shadow += currentDepth - bias > pcfDepth ? 1.0 : 0.0;
    }
}
shadow /= 9.0;
```

# Shadow aliasing

---

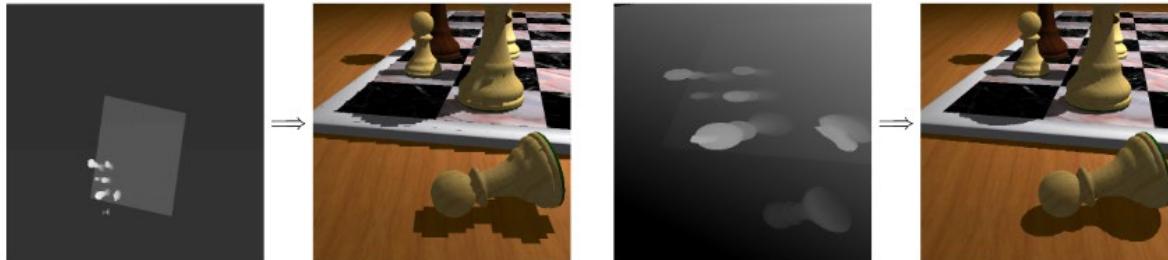
- Solution: sample from more than once from shadow map.



DigitalRune

# Shadow maps

## Improving Shadow maps



Perspective Shadow Maps  
[Stamminger and Drettakis, 2002]

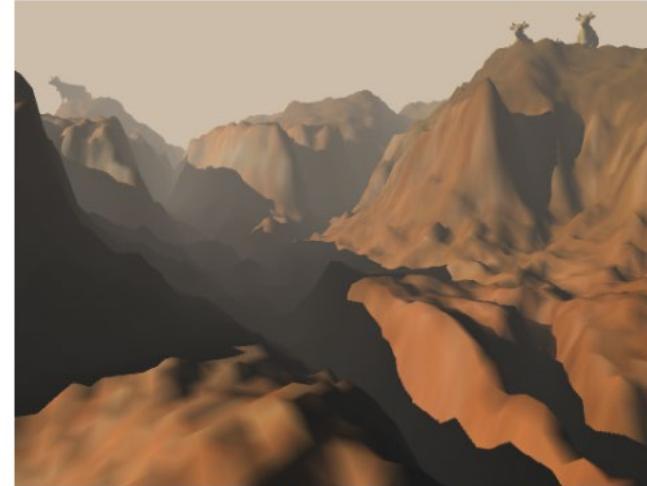


Figure 3-1. Large scale terrain rendering with 4-splits CSM

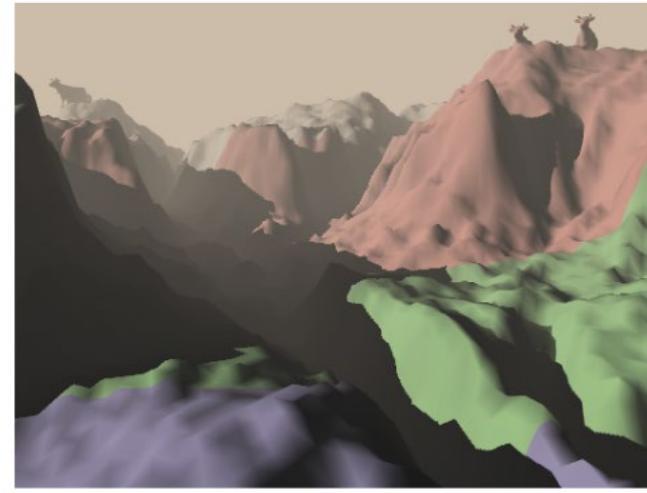


Figure 3-2. Texture look ups from different shadow maps are highlighted

Cascaded Shadow Maps [Dimitrov, 2007]

# Comparison

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- Shadow Volumes:
  - Pros: accurate hard shadows
  - Slower, rasterization heavy
- Shadow Maps:
  - Pros: Fast, supports soft shadows
  - Cons: high memory usage, aliasing